Mathematics 8

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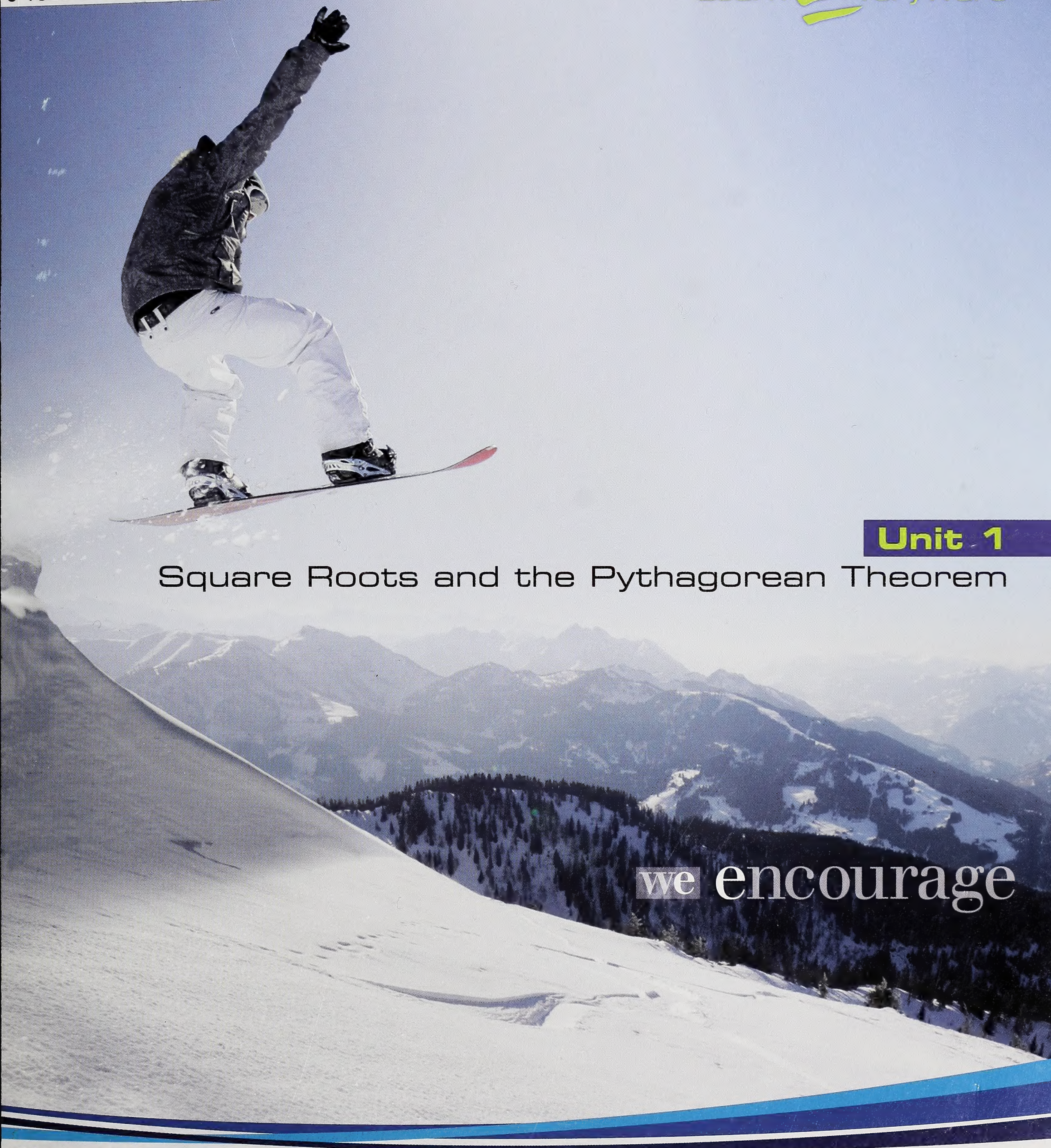

Unit 1

Square Roots and the Pythagorean Theorem

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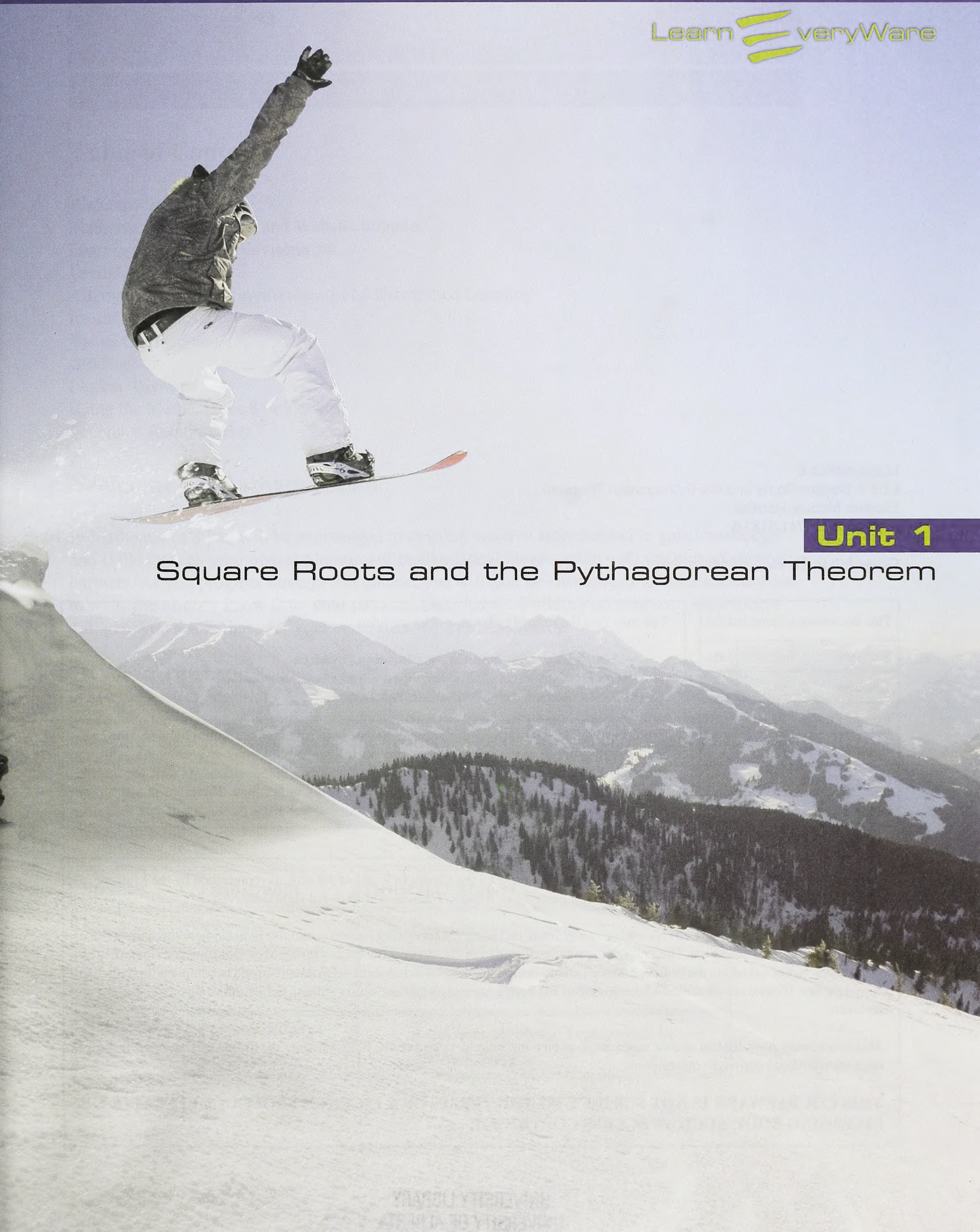

Mathematics 8
Unit 1: Square Roots and the Pythagorean Theorem
Student Module Booklet
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Education, **http://www.education.gov.ab.ca**
- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**
- Tools4Teachers, **http://www.tools4teachers.ca**

Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Mathematics 8 Introduction

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Welcome to Mathematics 8

In Mathematics 8 you will be encouraged to develop positive attitudes and to gain knowledge and skills through your own exploration of mathematical ideas—often with the help of study partners. As you progress through this course, you will also be encouraged to make connections to what you already know from your personal experiences. Building on your own experiences will give you a solid base for your understanding of mathematics.

The Alberta Program of Studies tells what students are expected to learn in mathematics courses. These expectations are written in statements called learning outcomes. The learning outcomes are organized into four stands that flow across programs from Kindergarten to Grade 9. Some of these strands are divided into smaller strands called substrands. Mathematics 8 will lead to further learning within these strands and substrands.

You can see the four strands (and substrands) and their general learning outcome in the following table.

Strands	Substrands	General Outcomes
Number		• Develop number sense.
Patterns and Relations Patterns	Patterns	• Use patterns to describe the world and to solve problems
	Variables and Equations	• Represent algebraic expressions in multiple ways.

Shape and Space	Measurement	• Use direct and indirect measurement to solve problems.
	3-D Objects and 2-D Shapes	• Describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.
	Transformations	• Describe and analyze position and motion of objects and shapes.
Statistics and Probability	Data Analysis	• Collect, display and analyze data to solve problems.
	Chance and Uncertainty	• Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Mathematics 8 Textbook and Website Support

You will be using MathLinks 8 as your textbook for this course. You will find additional support at the textbook's online website, *MathLinks 8*, at **http://highered.mcgraw-hill.com/sites/0070973385/student_view0/index.html**. Here, you can find tips for success in mathematics, master sheets, general web links, web games, and a course glossary. By choosing a chapter from the pull-down menu, you can access interactive quizzes and web resources for individual chapters.

LearnAlberta.ca

LearnAlberta.ca is a protected digital learning environment for Albertans. This Alberta Education portal, found at **http://www.learnalberta.ca/**, is a place where you can support your learning by accessing resources for projects, homework, help, review, or study.

For example, LearnAlberta.ca contains a large Online Reference Centre that includes multimedia encyclopedias, journals, newspapers, transcripts, images, maps, and more. The National Geographic site contains many current video clips that have been indexed for Alberta Programs of Study. The content is organized by grade level, subject, and curriculum objective. Use the search engine to quickly find key concepts. Check this site often as new interactive multimedia segments are being added all the time.

If you find a password is required, contact your teacher or school to get one. No fee is required.

Alternative Learning Environments and Distributed Learning

Distributed Learning is a model through which learning is distributed in a variety of delivery formats and mediums—print, digital (online), and traditional delivery methods—allowing teachers, students, and content to be located in different, non-centralized locations. Mathematics 8 students will be completing this course in a variety of learning environments, including traditional classrooms, online/virtual schools, home education, outreach programs, and alternative programs.

Instructional Design

Explanation

The learning model used in Mathematics 8 is designed to be engaging and to have you participate in inquiry and problem solving. You will actively interpret and critically reflect on your learning process. Learning begins within a community setting at the centre of a larger process of teaching and learning. You will be encouraged to share your knowledge and experiences by interaction, feedback, debate, and negotiation.

Components

This course uses the following structure and instructional design to connect you to the relevant curriculum and scientific concepts in Mathematics 8. These components are used throughout this course and will help you in seeing the context and overall content of the program.

Assignments - The Assignments indicate work that must be completed as your record of achievement. It may be a posting to a discussion board, an addition to your Math 8 folder, the completion of Lesson Question Set, or some other work assigned by your teacher.

Connect - In the Connect component of a lesson you are given opportunities to build on your understanding which you developed in the Explore part of the lesson.

Discuss and Share - Discuss and Share provides opportunities for you to interact with your peers and teacher. This may involve communication through a discussion board.

Explore - In the Explore component of a lesson you try to solve a somewhat open-ended problem in order to discover new concepts. You may be directed to work with a partner on problems in Explore.

Get Focused - The Get Focused introduces the lesson and focuses the unit inquiry to the level of lesson. This component includes the lesson introduction, learning objectives, list of materials needed, a critical question, and a list of assessments items.

Going Beyond - Going Beyond gives you the choice of challenging and enriching your knowledge and skills beyond the lesson.

Lesson - Each lesson consists of the main learning content from which you explore, reflect, and connect. The length of each lesson is defined by content that covers at least one measurable learning outcome.


Math 8 Folder - Periodically, you will come across a Math 8 Folder icon. This is just a reminder to you to store your completed work in your folder for future reference or storage until your teacher requests to see your work. You should be saving all of your completed assignments and notes to your Math 8 Folder. You can read more about setting up a Math 8 Folder in this Course Introduction.

My Guide - The My Guide is a feature that poses questions or hints as a teacher would in helping you complete a task. The questions or hints are meant to help you to discover an effective strategy rather than specifically telling you how to complete the task. Think of the answers to the questions as coming from you, a classmate, or someone on your discussion board.


Read - The Read component uses textual, or reading, material to get across information to you. The Read component directs you to read from your textbook.


Self-Check - Self-Check provides you with a set of questions that have answers provided so that you can mark them yourself. The Self-Check often involve practice questions.

Summary - There are course, unit, and lesson summaries. Each Lesson Summary summarizes the content and concepts developed in the lesson and answers the essential question posed in the Get Focused part of the lesson. Each Unit Summary presents you with a unit problem, describes what you have accomplished in the unit, gives you review questions to complete, and tells you about unit level testing. The courses summary sums up what you accomplished over the entire course.


Try This - The Try This part of a lesson presents questions or tasks for which no answers are provided. The Try This questions are often questions that may be answered correctly in more than one way. Your teacher may decide to mark these questions.

Unit - Each Unit consists of content developed around a general or major learning outcome. Units are made up of lessons and include the Introduction and Unit Summary.


Watch and Listen - The Watch and Listen part directs you to multimedia content for concepts that have a visual element. Typical multimedia content includes computer video and interactive Flash items.

Visual Cues (Icons)

You will see icons throughout the course. These icons are clues regarding the type of activity you are about to begin.

The icons and their meanings are given.


Assignment


Read


Try This


Self-Check


Watch and Listen


Discuss and Share


Math 8 folder

Glossary

The Glossary component of the course is an alphabetized list of terms and their definitions. The list is made up of important terms that have been introduced in the lessons of the course. Your textbook and the textbook website also have a glossary.

Lesson Answers

Answers for Self-Check questions and the Going Beyond question are provided in the Appendix.

The Lesson Question Set questions are assessment items that are to be marked by your teacher. The Try This questions may or may not be graded by your teacher. You should find out ahead of time which Try This questions your teacher intends to mark. For graded items, submit your work to your teacher.

Using the Mathematics 8 Folder

The Mathematics 8 folder serves as the organized collection of samples of your work in Mathematics 8. It gives you an ongoing record of your efforts, achievements, self-reflection, and progress throughout the course. When you want to show your friends or family what you've been learning, your work is all there for them.

Throughout the course, you will be asked to place your work in the Math 8 folder. If you are unsure of the process, your teacher will walk you through it. This folder may be an electronic folder on a server or a physical folder.

In addition to being able to show others what you have done, the course folder lets you see your progress. It lets you see how your knowledge, skills, and understandings are growing. It also lets you review and annotate work you have already completed. You may find your course folder useful in preparing for tests, quizzes, and your unit summary problem.

Periodically, you will be asked to share items from your course folder with your teacher. This is not always for grading, as often your teacher may use these items to learn more about you and your interests or as a way of tailoring other work assigned to you.

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Square Roots and the Pythagorean Theorem

Unit 1: Square Roots and the Pythagorean Theorem

Unit 1 Introduction

New homes are constructed in a variety of shapes and sizes. What starts as a pile of lumber usually ends up as a very useful and attractive combination of triangles, squares, and rectangles. You can identify each of these shapes in the photograph.

What you may not be aware of is the special mathematical relationship between the side lengths of special triangles and the measurements of these other figures. In this unit you are going to discover that relationship and use it to find answers in a variety of measurement situations.

You will use square tiles and grid paper to represent rectangles and squares and to find the areas of these figures. You will also use a calculator and computer programs to find areas and side lengths of squares, triangles, and rectangles.

In this unit you will learn to find the squares and square roots of whole numbers and to estimate square roots of whole numbers. Using these skills, you will apply the Pythagorean relationship to find missing dimensions of triangles and to solve problems such as determining whether a triangle is a right triangle.

This unit will help you answer the following critical question: How are properties of square numbers and the Pythagorean theorem used in identifying patterns, measuring lengths, and designing structures?

The new concepts and skills with square roots and the Pythagorean theorem will be presented in seven lessons.

Mathematics 8 LearnEveryWare

Unit 1: Square Roots and the Pythagorean Theorem

Lesson 1: Learning About Square Numbers and Area Models

Lesson 2: Understanding Squares and Square Roots

Lesson 3: Measuring Line Segments

Lesson 4: Estimating Square Roots

Lesson 5: Introducing the Pythagorean Theorem

Lesson 6: Exploring the Pythagorean Theorem

Lesson 7: Applying the Pythagorean Theorem

Keep up to your assignments as you go through this unit. Keeping your work organized, either electronically or in paper version, will help you to review and reference the lessons you have learned. This may be important to help you on tests or while you are working on unit problems. Be sure to talk to your teacher about when and how to submit your assignments.

In this unit there are a variety of assignments you will be asked to complete. Some of these include:

- posting to the discussion board
- adding samples of your work to your Math 8 folder
- completing sets of questions for each lesson
- solving a unit problem at the end of the unit
- completing evaluation pieces assigned by your teacher

Strategies for Success

In order to support your success in this unit, follow these strategies.

Strategy 1

Make a foldable study tool according to the detailed instructions on page 78 of your textbook. Although this activity may not be graded for marks, you will benefit from this tool. Keep these points in mind as you develop and use this study tool.

- As you are working through the lessons, add formulas, diagrams, examples, and vocabulary words.

- The foldable can serve as a quick reference guide and will help you save time when you are ready to study for your unit test.

Strategy 2

In this unit you will be referring to pages 76 to 117 of your textbook.

- Take time to flip through these textbook pages.

- Look at illustrations, margin features, and main titles to get a sense of where you will be going.

Strategy 3

Read your lessons and textbook materials carefully.

- Pay special attention to tables, and diagrams. They have information that will help you understand what you are reading.
- Read and reread material. Take time to understand it.
- Ask yourself: What is new material to me? What do I already know?
- Move ahead with confidence.

Strategy 4

At the end of numbered sections in the textbook, you will see “Math Link” sections that feature puzzles or questions involving squares, square roots, and the Pythagorean theorem. Completing these puzzles and answering these questions will help you successfully complete the assignments which follow them.

Unit 1 Problem

You will conclude this unit with a problem. While working through the unit, you will gain the knowledge and skills needed to solve this problem. This problem will ask you to develop a strategy to calculate the length of a stringer for a staircase and then to apply your knowledge in designing a step stool.

For a preview of the Unit Problem, look ahead to “Challenge in Real Life” on page 117 of your textbook.

Unit 1: Square Roots and the Pythagorean Theorem

Lesson 1: Learning Square Numbers and Area Models

Get Focused

Squares and rectangles can be seen in the design of many modern buildings. In this building the outside wall includes a grid of reflective glass. By using a grid, you can visualize the area of a surface easily.

In this lesson you will use grids to relate special numbers called square numbers—or perfect squares—to areas.

You will need 2-cm grid paper for this lesson. Go to the Math 8 Multimedia DVD, and open "2-cm grid paper." Use heavy-weight paper, if it is available.

This lesson will help you answer the following critical question: What is a square number, and how does it relate to area?

Assignments

Your assignment will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 1: Lesson 1 Question Set

Explore

When looking at the properties of numbers, it is sometimes difficult to understand what the numbers mean. Creating a picture or some other representation of the properties can help us understand what is being expressed. In this lesson you will investigate number properties by drawing and studying rectangles and squares.

Before you begin, it is important to have a strong understanding of what a rectangle is. When thinking about rectangles, it is easy to think of them as a four-sided figure, a quadrilateral, where two parallel sides are longer than the opposite parallel sides. A more accurate definition of a rectangle is "a quadrilateral that has four right angles."

Self-Check

SC 1. Is every square a rectangle? Explain.

Compare your answers in the Appendix.

Read

Who was Pythagoras, and what does he have to do with squares and rectangles? Read the top part of page 80 in your textbook down to "Explore the Math" to get an idea why his name is applied to the relationship central to this unit.

Try This

TT 1. Complete questions 1, 2, 3, 4, and 5 of "Explore the Math" on pages 80 and 81 of your textbook.

Draw out the figures, and write down the answers to the questions because you will be placing them in your Math 8 folder.

Pay special attention to the "Literacy Link" in each margin, so you are sure what is being asked. Having a strong understanding of words is important in knowing what to do in these lessons as well as on tests. Making the foldable shown on page 78 of your textbook to record definitions and key ideas as you encounter them is strongly advised. The link below will allow you to print out the graph paper if none is available.

Working with a partner in this activity may be beneficial, if one is available. Ask your teacher about the possibility of working with a partner.

If you have 36 square tiles available, you will not have to cut them out. Math tiles or Scrabble game tiles will work. If you need to make the square tiles required, print out a copy of 2-cm grid paper, using heavy-weight paper if it is available. Cut out 36 square tiles from the grid paper. Print a second sheet of grid paper on regular paper to record your shapes in the investigation. Be sure to label each shape as you make it so you will know later what it represents. Save the squares in an envelope for later use, if needed.

After you have completed "Explore the Math" and written answers for the questions and blanks in the tables, take a moment and go through My Guide to ensure you have a good understanding of what you discovered in your exploration.

My Guide

Q: What do the side lengths of a rectangle for a number represent?

A: The side lengths represent factors of the number. For example, if you have a rectangle with an area of 12, you could have drawn it on the grid paper with side lengths of 3 and 4. Notice how 3 and 4 are factors of 12.

Q: For some of the numbers, you could make square shapes on the grid paper. How does the side length of such a square shape relate to its area?

A: Take the number for the side length. Multiply this number by itself. The product gives the number for the area.

By making various rectangles, you will discover that some numbers can be represented on the grid paper by square shapes. Answer the following Try This questions.

TT 2. For which numbers were you not able to make square shapes on the grid paper?

TT 3. For which numbers were you able to make square shapes on the grid paper?

TT 4. You could make a square shape for some of the numbers. How does the side length of the square shape relate to the number?

TT 5. Suppose you had more than 36 tiles. Give a number of tiles that you think could be put together to make a square shape on the grid paper. Explain why you think this number would work.

Discuss and Share

Post your responses to the Try This questions on the discussion board. Then respond to at least two other postings.

Connect

In the Explore, you have discovered that some numbers can be represented by square shapes made up of unit tiles on a grid paper. The area of the square shape represents a **square number** or **perfect square**.

square number: a number that is the product of a whole number multiplied by itself

It is also known as a perfect square. For example, 36 is a square number because it is the product of the whole number 6 multiplied by itself.

perfect square: a number that is the product of a whole number multiplied by itself

It is also known as a square number. For example, 36 is a perfect square because it is the product of the whole number 6 multiplied by itself.

Read

Turn to page 82 of your textbook and read and work through "Example 1: Identify Perfect Squares." My Guide is there to help you work through examples.

My Guide

Q: In Example 1.a), the factors chosen to begin the factor tree for 24 could have been 3 and 8. Would the final factors still be the same?

A: Yes, the prime factors would still end up the same.

Note: In perfect squares, the prime factors can always be arranged in two groups that have exactly the same numbers. You can see this in Example 1.c).

Try This

TT 6. Should 1 be considered a square number? Why or why not?

TT 7. Suppose you know the area of a square shape. The number for the area is a perfect square.

a. Explain how you would find the side length of the square shape.

b. Explain how you would find the perimeter of the square shape.

Place a copy of your answers in your Math 8 course folder.

Read

What are the main points you learned so far? Review the information in the margin on page 82 of your textbook. Now may be a good time to make a foldable and add some definitions and key ideas to it.

Self-Check

SC 2. Do "Check Your Understanding" questions 5, 7, 17, 21, and 23 on pages 85 and 86 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Turn to pages 85 to 87 in your textbook. For extra practice, you may complete questions 6, 8, and 22. Then check your work using the shortened answers given on page 484 at the back of your textbook.

Assignment

To inform your teacher of your progress, this question set will need to be completed and submitted to your teacher.

As you complete these questions, you may wish to answer electronically instead of as a paper copy. To assist you with this, go to the Math 8 Multimedia DVD and read “MS Word” for more information about building diagrams and shapes using your computer.

Go to the Unit 1 Assignment Booklet, and complete “Unit 1: Lesson 1 Question Set.”

Going Beyond

So far, you have used rectangular shapes and square shapes to help understand square numbers and square roots. Looking for patterns in numbers and shapes helped Pythagoras develop an understanding of the world. We can also benefit by developing our ability to see patterns. What can we learn from triangular shapes? What patterns there can help us? See for yourself by building a triangle of triangles and looking for patterns.

Turn to page 87 of your textbook and complete “Extend” question 24.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you represented a square number (i.e., perfect square) as a square figure. You also determined whether or not a number is a square number by using square tiles to try and construct larger squares to represent the number, using grids to try and draw squares to represent the number, and factoring the number.

You were able to conclude that a square number can be represented by a square figure. With the area of this region representing the square number, the side length of the square will be a whole number.

Unit 1: Square Roots and the Pythagorean Theorem

Lesson 2: Understanding Squares and Square Roots

Get Focused

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Some buildings have patios in which the surface is made of square tiles. If the patio is a square surface, then the number of tiles will be a square number. You can determine whether any number is a square number by looking at its factors. You may remember that a factor will divide exactly into the number.

Suppose you know that a square patio is made up of 16 tiles. This observation will then allow you to know that 16 is a square number and it will have a set of equal factors (4 x 4). If you know your local skateboard park has a square patio area made up of 9801 tiles, then you will be able to make an inference about the factors of 9801.

In this lesson you will discover ways to identify square numbers and to produce square numbers. As well, starting with any square number, you will find the number that can be multiplied by itself to produce that square number. For example, starting with the square number 81, you will find the number that can be multiplied by itself (in this case, 9) to produce the square number (81).

This lesson will help you answer the following critical questions:

- What do factors indicate about square numbers?
- Which factor of a square number is its square root?

Assignments

Your assignment will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 1: Lesson 2 Question Set

Explore

You made rectangles for numbers in Lesson 1. Making rectangles for numbers gave you information about how the numbers work. This information is called the properties of the number. Properties of a number can also be revealed from the list of **factors** of the number.

factor: a number that exactly divides a given number

For example, 6 is a factor of 12 since $12 \div 6 = 2$ with a remainder of 0. However, 8 is not a factor of 12 since $12 \div 8$ has a remainder of 4.

Read

Any given number greater than 1 has at least two factors but may have many more. Look at the factor trees in “Example 1: Identify Perfect Squares” on page 82. These show all the prime factors of each number, except 1 and the number itself. Those are left out because it is already known and that extra information interferes with what the factor tree can explain.

Self-Check

SC 1. What are all the factors of 10?

SC 2. What are all the factors of 24?

Compare your answers in the Appendix.

What does a list of factors tell you about a given number? You began to develop answers to this question in Lesson 1 as you replied to “Explore the Math” on pages 80 and 81 of your textbook. Review those questions and answers now.

Write your answers to the following Try This questions.

Try This

TT 1. What does a list of factors tell you about a given number?

Working with a partner in this activity may be beneficial, if one is available. Ask your teacher about the possibility of doing so.

The following activity will allow you to find whether a given number has an odd or even number of factors and what that information means.

Go to the Math 8 Multimedia DVD, and open “Factor Chart.” You may print out a copy of “Factor Chart” to help you.

You can determine the factors of each number to fill in the chart using factor trees as you did in the previous lesson. However, the Factor Finder will generate the factors of these and other numbers, and it may be interesting to use it. When you choose the word “Factor” by clicking on it, definitions and a demonstration applet will appear. Study the natural numbers definition of factor, which appears on the screen, including the examples, so you are clear on how the words are used. This may be a useful definition to add to your foldable.

You can insert the first 30 natural numbers, in turn, into the first text box and the factors will be calculated for you so that you can fill in your chart.

You can use the number of factors to tell what kind of number you are working with. There are differences between numbers with two factors—an odd number of factors and an even number of factors. You may want to look again at the “Literacy Links” in the margins on pages 80 to 82 of your textbook to refresh your recall of the definitions.

Try This

TT 2. What kind of number has only two factors?

TT 3. What kind of number has an odd number of factors?

TT 4. What kind of number has an even number of factors?

Discuss and Share

Post your responses to TT 2, TT 3, and TT 4 on the discussion board. Then respond to at least two other postings.

Connect

In Lesson 1 you discovered that if the prime factors from a factor tree can be arranged in two identical groups, the number is a perfect square. In the Try This activity just completed, you discovered that the list of factors of a given number indicates whether the given number is a square number or not. But there is still another way of telling if a given number is a square number.

In the Demonstration Applet, the second text box used division to see if a particular number was a factor of the number in the first text box. The division sentence may written as

$$\frac{25}{5} = 5 \text{ or } 25 \div 5 = 5$$

A division sentence involves a **dividend**, **divisor**, and **quotient**. If the divisor and the quotient are equal, the quotient is a square number. For example,

$$\begin{array}{r} \text{dividend} \longrightarrow \\ \text{divisor} \longrightarrow \end{array} \frac{25}{5} = 5 \longleftarrow \text{quotient}$$

or

$$\text{dividend} \longrightarrow 25 \div \underset{\substack{\uparrow \\ \text{divisor}}}{5} = 5 \longleftarrow \text{quotient}$$

Notice how the divisor and the quotient are equal (e.g., both 5); that tells us that 25 is a square number.

dividend: the name given to a number which is to be divided by another number

For example,

$$\text{dividend} \rightarrow 25 \div 5 = 5$$

divisor: the name given to a number which is used to divide into another number

For example,

$$25 \div \underset{\substack{\uparrow \\ \text{divisor}}}{5} = 5$$

quotient: the name given to a number which is the result of division

For example,

$$25 \div 5 = 5 \longleftarrow \text{quotient}$$

square root: a number that when multiplied by itself results in a given number

A pair of factors for a given number corresponds to the side lengths of a rectangle you can draw on a grid for the given number, as you did in Lesson 1. If the rectangle is a square, then the side length is the **square root** of the number.

If you look at the table of factors for the first 30 numbers that you completed earlier in this lesson, each of the perfect squares has an odd number of factors. When the factors are arranged from smallest to largest, the middle factor is the square root of the number. Check to prove that this statement is true for 4, 9, 16, and 25.

Read

Turn to page 83 of the textbook. Then carefully follow "Example 2: Determine the Square of a Number" and "Example 3: Determine the Square Root of a Perfect Square," including Method 3 on page 84. See how to square a number and find its square root. See how listing factors can help you find square roots and identify square numbers. Pay close attention to the definitions, the "Literacy Links," and "Tech Link" in the margins.

Watch and Listen

Go to the Math 8 Multimedia DVD, and open "Exploring Square Roots—Use It." Find 13 perfect squares, and see their area on a grid and their square roots with "Exploring Square Roots—Use It." This program asks you to manipulate the size of an area to match a perfect square and links side length of the square to the square root.

Remember that a correct symbol for the square root of any number would be $\sqrt{(\text{any number})}$.
In the previous example, it would be written as $\sqrt{25} = 5$ and read as "the square root of 25 equals 5."

You have seen how you find the square root of a number. The **inverse operation** of finding the square root of a number is **to square** a number.

inverse operation: an operation that undoes the action of another operation

For example, subtraction is the inverse operation of addition.

to square: to multiply a number by itself

Try This

TT 5. In helping her partner, Theresa said, "Finding the square root of a number and squaring a number are inverse operations." But her partner did not understand the concept of inverse operations. Some examples of inverse operations may help Theresa's partner understand the relationship between finding a square root and squaring a number. What examples of inverse operations can you think of?

TT 6. In a list of the factors of a perfect square, one factor is a square root but not the others. Why is this true?

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 3. Do "Check Your Understanding" questions 9, 11, 13, 14, 15, and 18 on pages 85 and 86 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Turn to pages 85 and 86 in your textbook. For extra practice, you may complete any or all of questions 10, 12, 16, and 22.a), and 22.b) of "Check Your Understanding." Then check your work using the answers given on page 484 of your textbook.

Assignment

To inform your teacher of your progress, this question set will need to be completed and submitted to your teacher. As you complete these questions, you may wish to answer electronically instead of on a paper copy. To assist you with this, you may wish to review "MS Word" on the Math 8 Multimedia DVD for more information about building diagrams and shapes using your computer. You may also wish to review "Factor Finder."

Go to the Unit 1 Assignment Booklet, and complete "Unit 1: Lesson 2 Question Set."

Going Beyond

When you add two even numbers, the sum is also an even number. What happens when you add two square numbers? Is the sum also a square number?

Turn to page 87 of your textbook, and complete "Extend" question 27. If you don't have a calculator, there is probably one in your computer. For a PC, click "Start," go to "All Programs," and then select "Accessories." When you choose "Calculator," make sure the "View" is standard. The square root button may be labelled "sqrt."

Compare your answers in the Appendix.

Lesson Summary

At the beginning of this lesson, you looked at tiles on a square patio and at determining a square number by counting the number of tiles. In this lesson you found two new ways to produce square numbers and to identify them. You were shown that a number that has an odd number of factors is a perfect square, but a number that has an even number of factors is not a perfect square. You also found that a perfect square is formed when a number is multiplied by itself.

You became acquainted with square roots. You found that the square root of a number is a smaller number that has to be multiplied by itself to produce the original larger number. Therefore, if a number is divided by a smaller whole number and the divisor and quotient are equal, the divisor is the square root of the original number and the original number is a perfect square. You also saw that when the factors of a perfect square are listed in ascending order, the middle factor is the square root.

You saw that squaring a number and taking the square root of a number are inverse processes.

$$\sqrt{16^2} = 16 \text{ and } \left(\sqrt{16}\right)^2 = 16$$

Unit 1: Square Roots and the Pythagorean Theorem

Lesson 3: Measuring Line Segments

Get Focused

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The area of solar panels—rather than the length or width—is the most important measurement. The area affects how much energy from the Sun can be collected and converted to electricity.

Sometimes you will draw squares and use their area to find other measurements. This lesson will help you answer the following critical question: How can you use the area of a square figure to find the length of a line segment?

You will need 1-cm grid paper for this lesson. Go to your Math 8 Multimedia DVD, and open "1-cm grid paper."

Assignments

Your assignment will consist of the following:

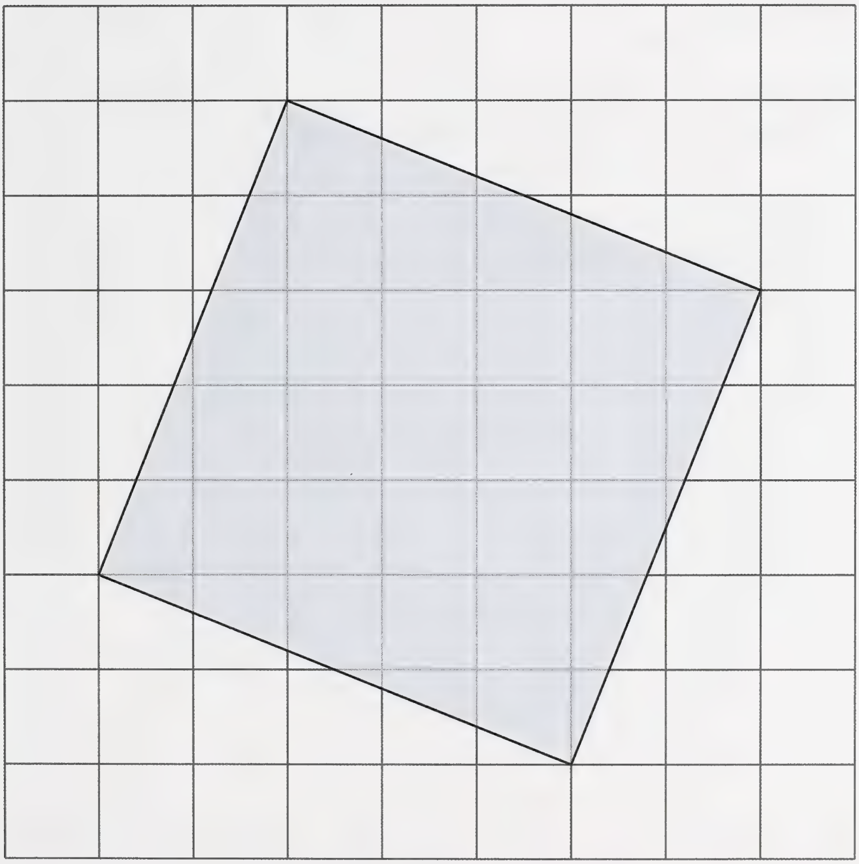
- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 1: Lesson 3 Question Set

Explore

When the sides of a square fall on the lines of the grid, the side length of the square is easy to see. When a square is at an angle on a grid, you must use a new strategy to find the side lengths.

Try This

TT 1. Find the side length of the square below that sits at an angle to a grid. Call it square B for blue. Approach the problem by first finding the area of square B.



Working with a partner in this activity may be beneficial, if one is available. Ask your teacher about that possibility.

You may print out a copy of the 1-cm grid paper with tilted square.

Use the following two strategies to find the area of blue square B and to help relate side length to area.

Strategy 1

Find the areas of regular figures within the tilted square and add them together to find the area of square B. Find the side length of square B by taking the square root of the area of square B.

It sounds complicated, so take it one step at a time. By the end of the lesson you will be saying, “That wasn’t as tough as I thought.”

Draw the largest possible square within square B that follows the grid lines and stays within square B. Call it square A. Find the area and side length of square A.

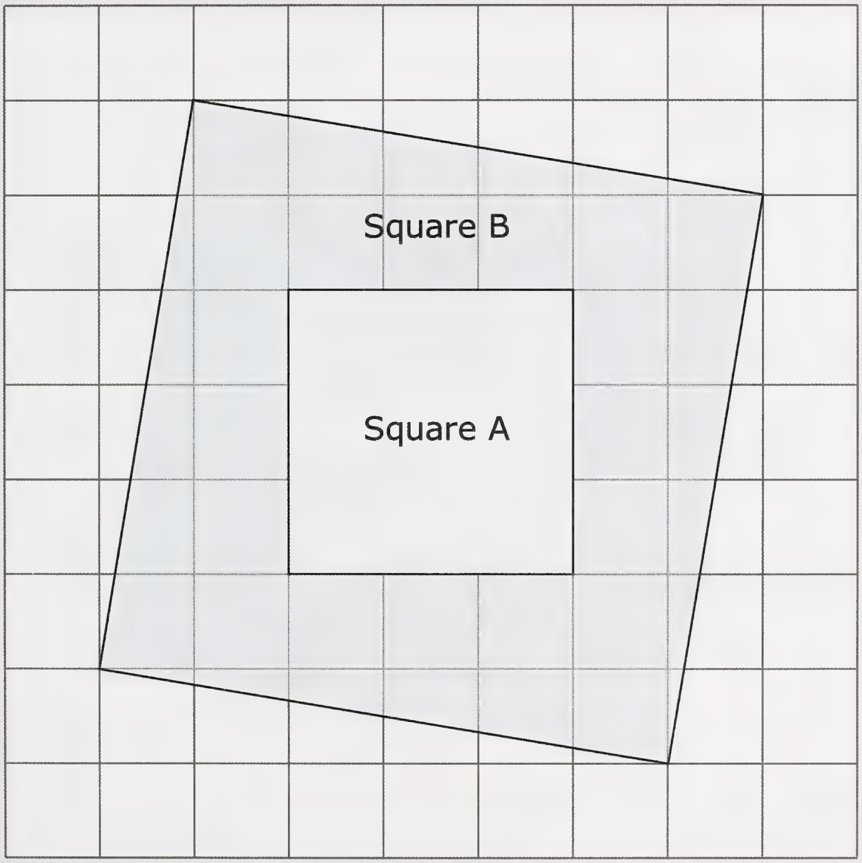
My Guide

Q: What is the area of square A?

A: The area of square A is 9 square units.

Q: How is the side length of square A related to its area?

A: The side length found by taking the square root of 9. So the side length of the square is equal to $\sqrt{9}$ units or simply 3 units. We can see this makes sense since the area of a square is the side times the side, and $3 \times 3 = 9$.



Construct right triangles around the central white square to fill the rest of the blue square. Find the areas of these triangles.

My Guide

Q: What is the combined area of the orange triangles?

A: If the lower orange triangle was placed directly above the upper orange triangle, the two orange triangles would form a rectangle 2 squares high and 5 squares wide. Therefore, the total area of the orange triangles is 10 unit squares. Alternately, you can calculate the area of each triangle as follows:

$$\begin{aligned}\text{Area} &= \tfrac{1}{2}\,\text{base} \times \text{height}\\ &= \tfrac{1}{2} \times 5 \times 2\\ &= 5\end{aligned}$$

Therefore, the total area of the two orange triangles is 2 x 5 = 10 unit squares.

Q: What is the combined area of the green triangles?

A: If the upper green triangle was placed directly to the right of the lower green triangle, the two green triangles would form a rectangle 2 squares wide and 5 squares high. Therefore, the total area of the green triangles is 10 unit squares. Alternately, you can calculate the area of each triangle as follows:

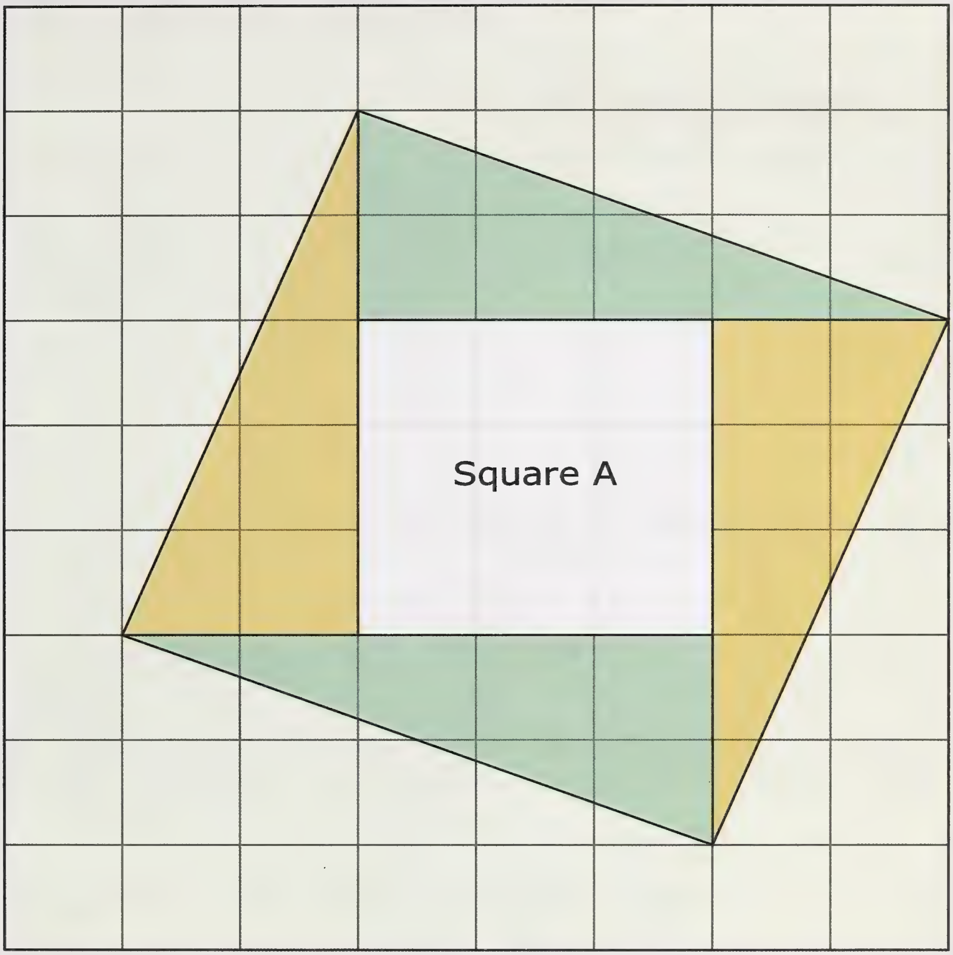
$$\begin{aligned}\text{Area} &= \tfrac{1}{2}\,\text{base} \times \text{height}\\ &= \tfrac{1}{2} \times 5 \times 2\\ &= 5\end{aligned}$$

Therefore, the total area of the two green triangles is 2 x 5 = 10 unit squares.

Q: Are the green triangles and the orange triangles all the same size?

A: Yes, each of the triangles is the same size, regardless of colour or position. Therefore, the total area of the triangles can be found by finding the area of one triangle using the formula $\text{area} = \tfrac{1}{2} \times \text{base} \times \text{height}$ and then multiplying by 4.

Q: Why use green triangles and the orange triangles if they are all the same size?



A: Using colours enables us to visualize the areas when they add to form rectangles, instead of just relying on a formula.

Find the total area of square B. Then take the square root of the area of the square B to find the side length.

My Guide

Q: What is the area of the blue square B?

A: The blue square B is exactly covered by the white square and the orange and green triangles. So the area of the blue square B is equal to the combined areas of the white square and the orange and green triangles. The orange triangles have a combined area of 10 square units, and the green triangles have a combined area of 10 square units. There are also 9 whole grid squares within square A. So the area of the blue square B is 10 unit squares plus10 unit squares plus 9 unit squares, which totals 29 square units.

Q: What is the side length of the blue square?

A: The area of the blue square B is 29 square units. Therefore, the side length of the blue square B is the square root of 29 square units or $\sqrt{29}$ units. If the blue square B is drawn on a 1-cm grid, then the side length is $\sqrt{29}$ cm. Since $\sqrt{25} = 5$ and $\sqrt{36} = 6,$ the value of $\sqrt{29}$ must be between 5 and 6, but slightly closer to 5. We can estimate the side length of square B to be approximately 5.4 units.

Looking at the length of any side of square B and comparing it to the grid lines shows that a length of 5.4 units is reasonable.

There is another way to find the side length of blue square B.

Try it and see if you like it better. This will also double check the answer found in Strategy one.

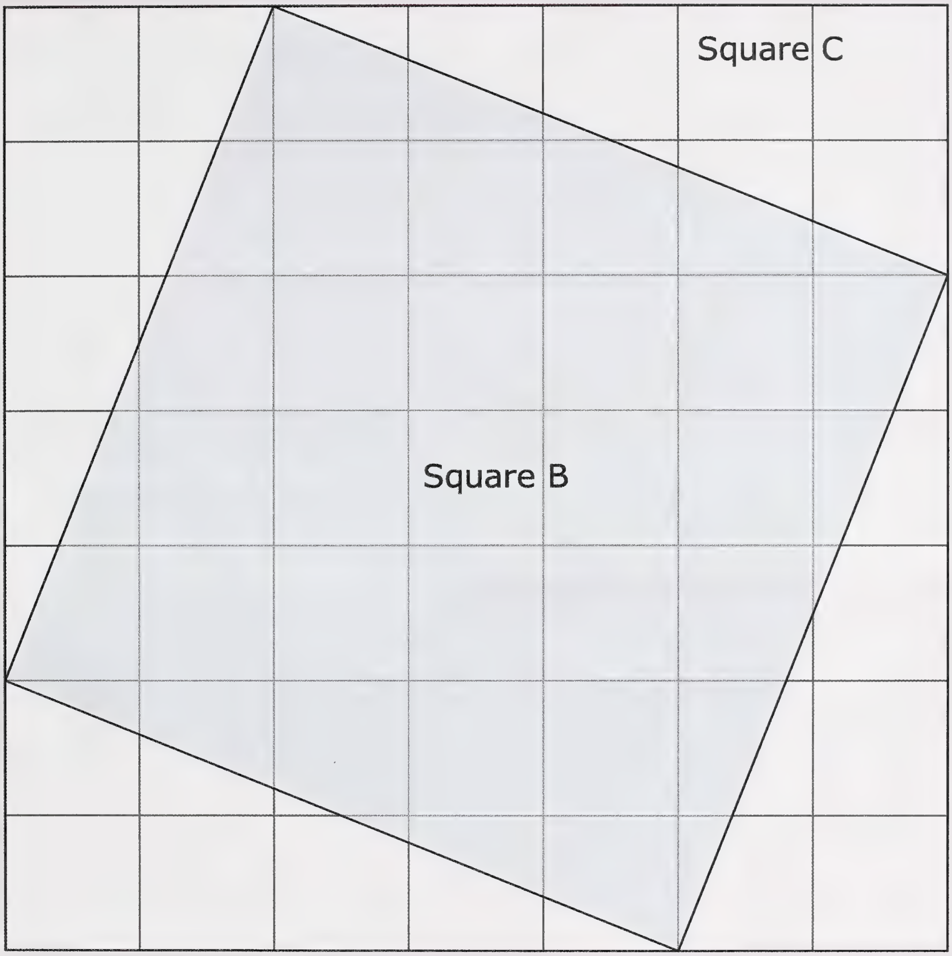
Strategy 2

Find the areas of regular figures outside the tilted square B, and subtract them to find the area of the blue square B. Find the side length of square B by taking the square root of the area of square B.

Draw the smallest possible square outside square B that follows the grid lines and stays without the blue square. Call it square C. Find the area of square C.

My Guide

Q: What is the area of square C?



A: The area of square C is 49 square units, because each side length is seven units.

Observe the right triangles around the central tilted square that fill the rest of the large square. Find the combined areas of these triangles.

My Guide

Q: What is the combined area of these triangles?

A: If the upper right triangle was placed directly above the lower left triangle, the two opposite triangles would form a rectangle 2 squares high and 5 squares wide. Therefore, the total area of these two triangles is 10 unit squares. Similarly, if the upper left triangle was placed directly to the left of the lower right triangle, these two opposite triangles would form a rectangle 2 squares wide and 5 squares high. Therefore, the total area of the two triangles is also 10 unit squares. Thus, the combined area of all four triangles is 20 unit squares. Alternately, you can calculate the area of each triangle as follows:

$$\begin{aligned}\text{Area} &= \frac{1}{2}\ \text{base} \times \text{height}\\ &= \frac{1}{2} \times 5 \times 2\\ &= 5\end{aligned}$$

Therefore, the total area of all four triangles is $5 \times 4 = 20$ unit squares.

Find the total area of square B using subtraction, and then take the square root of the area of square B to find the side length.

My Guide

Q: What is the area of the square B?

A: The area of square B is found by subtracting the combined areas of the yellow triangles from the area of square C. So the area of square B is 49 unit squares minus 20 unit squares, which totals 29 unit squares.

Q: What is the side length of square B?

A: As in the previous strategy, the area of square B is 29 square units. Therefore, the side length of square B is the square root of 29 square units or $\sqrt{29}$ units. If the blue square B is drawn on a 1-cm grid, then the side length is $\sqrt{29}$ cm. Since $\sqrt{25} = 5$ and $\sqrt{36} = 6$, the value of $\sqrt{29}$ must be between 5 and 6, and probably closer to 5. We can estimate the side length of square B to be approximately 5.4 units.

Looking at the length of any side of square B and comparing it to the grid lines shows that a length of 5.4 units is reasonable.

TT 2. List the areas and side lengths for squares A, B, and C.

TT 3. How did the side lengths of the squares you drew compare to the area of the squares?

TT 4. Which of the two strategies do you prefer? Explain why.

Discuss and Share

Post your responses to TT 2, TT 3, and TT 4 questions on the discussion board. Then respond to at least two other postings.

Connect

You have discovered that the side length of a square can be found from its area.

Try This

TT 5. In talking with his partner, Lin asked, “Is the area of every square a square number?” Provide an answer to Lin. In your answer, include a drawing or a description of squares in your answer.

TT 6. When will the length of a line segment not be a whole number?

Place a copy of your answers in your Math 8 course folder.

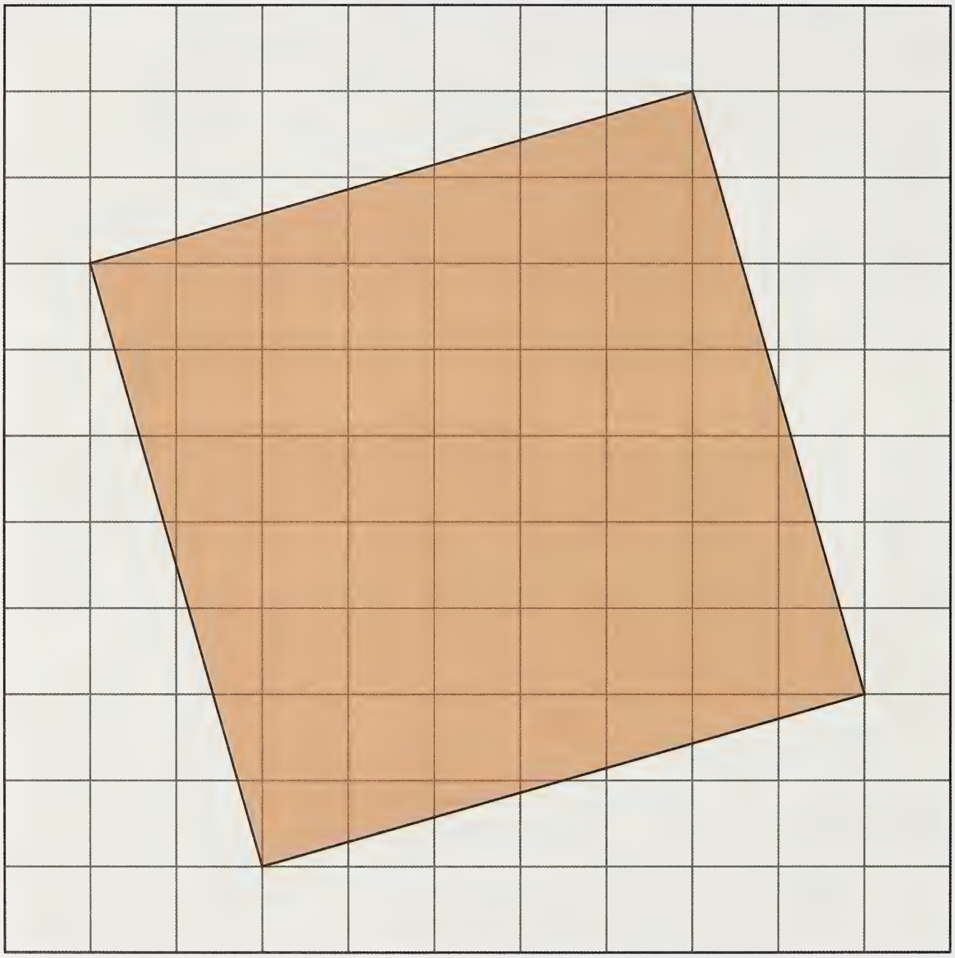
Read

Now would be an excellent time to review “Key Ideas” on page 84 of your textbook and update your foldable, if you have one. If you haven’t started one, perhaps now would be a good time to do so.

Self-Check

SC 1. Do “Apply” questions 19, 20, and 22.c) on page 86 of your textbook.

SC 2. a. For the square below drawn on 1-cm grid paper, find the area of the orange square.

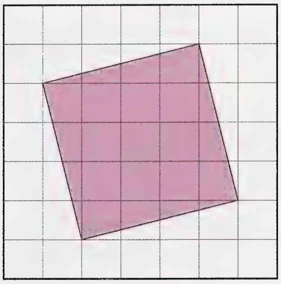


b. Find the side length of the orange square.

Compare your answers in the Appendix.

Extra Practice

Find the side length of the coloured square below using one strategy. Then check your work using the other strategy. The square is drawn on a 1-cm grid. Your final answer should have a numerical value of the square root of the square root of (300 – 11). You may print a copy of the coloured square below. Go to the Math 8 Multimedia DVD, and click on "Coloured Square."



Compare your answers in the Appendix.

Assignment

To inform your teacher of your progress, this question set will need to be completed and submitted to your teacher. You may find "MS Word" on the Math 8 Multimedia DVD useful in helping you draw shapes electronically. The file titled "Square on a Grid" on the Math 8 Multimedia DVD is also recommended for your help.

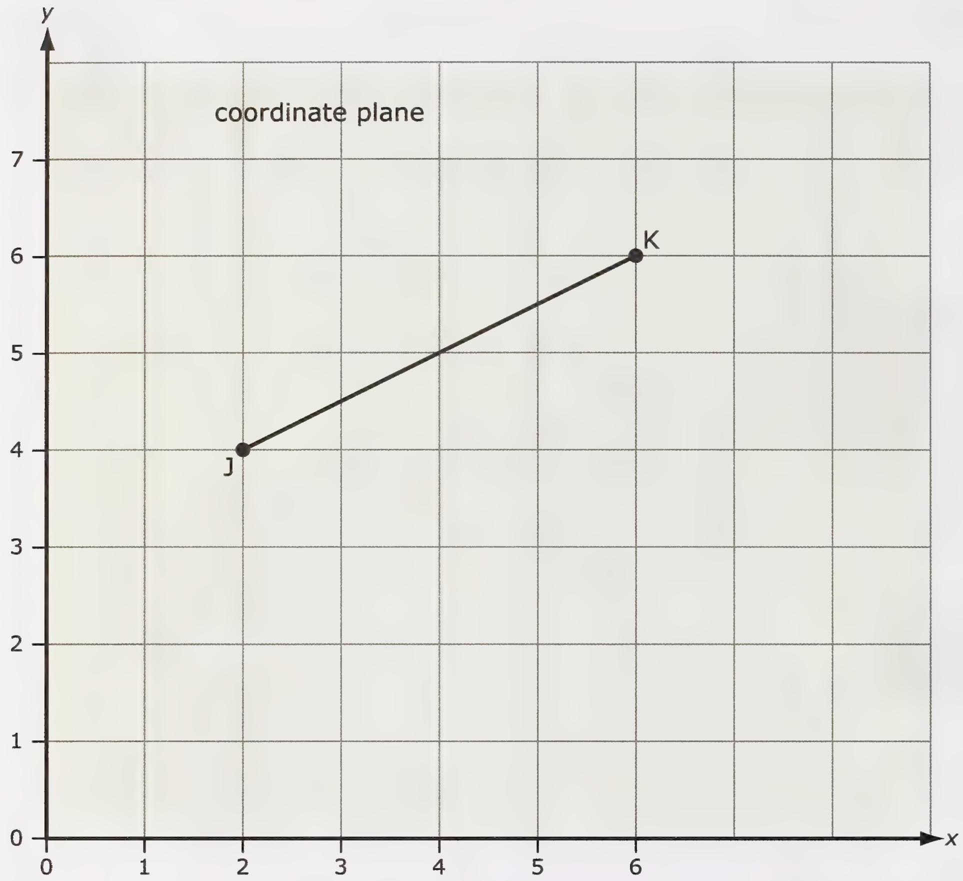
Go to the Unit 1 Assignment Booklet, and complete "Unit 1: Lesson 3 Question Set."

Going Beyond

You can use area to find the distance between two points on a coordinate grid. You will be working with points on coordinate grids more in a following unit.

Point J at (2, 4) and point K at (6, 6) are plotted on the coordinate plane below. What is the length of line segment $\overline{JK}$ joining the two points?

Hint: Construct a square on the coordinate grid with line segment $\overline{JK}$ as one side of the square. Then you will have a square at an angle to the grid. Use a method you learned in this lesson to find the length of one side of that square and you will have the answer.



Compare your answers in the Appendix.

Lesson Summary

In this lesson you used the area of a square—not to calculate the efficiency of a solar panel, but to find the length of a line segment equal to a side of the square. You were able to do this for a square that sits at an angle to a grid by using mathematics instead of just counting squares.

To find the side length of a square that sits at an angle to a grid, you drew a second square on the grid lines either inside the first square or enclosing the first square. You used the area of the second square and the area of surrounding triangles to find the area of the first square. The square root of this area gave you the length of the line segment.

A question was posed at the start of this lesson: How can you use the area of a square figure to find the length of a line segment? To answer that, you first find the area of the square figure of which the line segment is a side. The square root of this area gives you the length of the line segment.

Unit 1: Square Roots and the Pythagorean Theorem

Lesson 4: Estimating Square Roots

Get Focused

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Squares can be found in the parks of public buildings. Recall that the side length of a square is the square root of the number representing the area of the square. With an area of 100 m^2, the length of each side would be 10 m. If flowers were placed on the four sides, there would need a row of flowers 40-m long. When the area of a square is not a perfect square, the side length can only be approximated.

In this lesson you will determine the approximate square root of numbers. You will also use a handheld calculator or a computer to solve problems involving square roots. In addition to a calculator, you will need 1-cm grid paper. You can find "1-cm grid paper" on the Math 8 Multimedia DVD.

This lesson will help you answer this following critical question: How can you estimate the square root of a number that is not a square number (perfect square)?

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 1: Lesson 4 Question Set

Explore

In previous lessons, you have used the connection between the square root and the side length of a square. You know that the square root of the area of a square gives you the length of each of the four sides of that square.

You've been able to find the exact square roots for many numbers. Thinking of a square root as the side length of a square gave you a good strategy for finding the square root of a number.

Try This

TT 1. So why would you ever have to estimate to find the square root of a number? Find out by doing questions 1, 2, 3, 4, and 5 of "Explore the Math" on page 95 of your textbook.

Working with a partner in this activity may be beneficial, if one is available. Ask your teacher about the possibility of doing so.

You may find it helpful to answer this question by going to the Math 8 Multimedia DVD for grid paper.

You may also find "Exploring Square Roots—Explore It" helpful in placing square roots on the number line. You can find "Exploring Square Roots—Explore It" on the Math 8 Multimedia DVD.

My Guide

Q: What are the side lengths of the largest and the smallest of the square mats?

A: The side lengths of the mats are 7 m and 6 m. These were calculated by taking the square roots of the areas of the mats.

Q: What are the missing numbers in the boxes on the number line?

A: The following information should help with the number line:

$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{49} = 7$
$\sqrt{81} = 9$

Q: What is your estimate of the side length of the middle mat to one decimal place?

A: The value is between $\sqrt{36}$ m and $\sqrt{49}$ m or between 6 m and 7 m. The value should be the square root of the number chosen in question 1 of "Explore the Math." For instance, if the estimate for the area of the middle mat is 44 m^2, a reasonable estimate of the side length is 6.6 m. This is because 44 is closer to 49 than to 36, so $\sqrt{44}$ is closer to 7 than to 6.

TT 2. Draw a number line and show or describe where the following square roots fit on the number line:

$\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$, $\sqrt{18}$, $\sqrt{24}$

TT 3. What strategies did you use to estimate the values of the square roots?

TT 4. How would you use your calculator to check that your estimates for the square roots were correct?

Discuss and Share

Post your responses to TT 2, TT 3, and TT 4 on the discussion board. Then respond to at least two other postings.

Connect

Just like a tree grows out of a root because the root multiplies its own nutrients from the soil, a square number grows out of a square root when the square root is multiplied by itself.

Read

Read “Example 1: Estimate the Square Root of a Number” on page 96 of your textbook. Look to see how perfect square numbers and a number line are used to estimate square roots of a given number. When problem solving, you can use estimation to narrow down the numbers that you need to guess and test.

Self-Check

SC 1. Which square numbers are most helpful in estimating $\sqrt{28}$?

SC 2. Which square numbers are most helpful in estimating $\sqrt{350}$?

SC 3. Do “Check Your Understanding” questions 4, 10, 12, and 15 on pages 99 and 100 of your textbook.

Compare your answers in the Appendix.

You can find the square root of a number using a handheld calculator that has a square root button.

$\sqrt{}$

Remember, there is a calculator in your computer. For a PC, click “Start,” and go to “All Programs.” Then choose “Accessories.” When you choose “Calculator,” make sure the “View” is standard. The square root button may be labelled “sqrt.”

For a perfect square number, the calculator can display the exact square root. For other numbers—numbers that are not perfect square numbers—the calculator can only display an approximate value for the square root. That's because the square roots of these numbers cannot be expressed exactly by a decimal. In these cases, it would be appropriate to indicate that it is an approximate value. For example,

$\sqrt{25} = 5$ but $\sqrt{26} \approx 5.196$

Refer to your calculator manual for directions on using your calculator to find the square root of a number. On some calculators, you enter a number first and then press the square root button. For example, to find the square root of 4, you enter 4 and then press the square root button to get its square root of 2.

On other calculators, you press the square root button first, the number second, and finally the equal or enter button. For example, to find the square root of 25, you press the square root button, enter 25, and press the enter or equal button. These key strokes should give you the number 5 on the display.

Read

See the "Tech Link" in the margin on page 84 of your textbook for more tips on using a handheld calculator to determine square roots.

Try This

TT 5. You can use a handheld calculator or the calculator in your computer to find square roots. If you have a spreadsheet in your computer, such as "Excel" or "Microsoft Office Spreadsheet," you can also use it to calculate square roots.

Open the spreadsheet and, in any rectangle, type the following:

=sqrt(n)

Put a number in place of n.

When you press “Enter,” the square root of the number will be automatically calculated. To calculate the square root of a different number, put the different number inside the bracket in the formula bar rectangle shown above B and C near the top of the screen, and press “Enter.”

You can set it up to automatically calculate square roots and squares of any number. Here’s how:

- Find and open the spreadsheet, if you haven’t already.
- In column A and row 3, type Number.
- In column B and row 3, type Square Root.
- In column C and row 3, type Square.
- In column B and row 4, type =SQRT(A4). This tells the computer to take the square root of whatever number is in column A and row 4.
- In column C and row 4, type =A4*A4. This tells the computer to multiply the number in column A and row 4 by itself and to calculate the square of that number.
- Save the file as “Square Root Calculator.”

Now try putting the number 25 in rectangle A4. When you press “Enter,” “Tab,” or one of the arrow keys, the calculations will be made for you. Did it work? If not, review the steps carefully and make sure you follow them exactly.

Next, try a number that is not a perfect square, such as 8 or 32. If you want more decimal places to be displayed, widen the square root column by placing your cursor over the right-side edge of square B at the top of the screen until the two-way arrow appears. Then hold down the mouse button and drag the column wider. Usually, no more than 10 digits will be displayed.

If you want the computer to display more than one square root and square number at a time, just highlight and copy row 4 into row 5 and row 6. Then the numbers you place in A5 and A6 will also be calculated and displayed when you press "Enter."

Save any changes: You can select the save button or press control ("Ctrl") and the "S" key simultaneously. Name the file something like "My Square Root Calculator." In lessons that involve taking square roots of several numbers or squaring several numbers, this program may be more convenient than the calculator.

Read

How can you find a number that has a square root between two numbers? Read "Example 2: Identify a Number With a Square Root Between Two Numbers" on page 97 and "Key Ideas" on page 98 of your textbook.

Self-Check

SC 4. Do "Communicate the Ideas" questions 2 and 3 on page 98; then complete "Check Your Understanding" questions 6, 8, and 14 on pages 99 and 100 of your textbook.

Compare your answers in the Appendix.

Extra Practice

For extra practice, you may complete questions 5, 7, 9, 13, 16, and 17 from "Check Your Understanding" on pages 99 and 100 of your textbook. Then check your work using the shortened answers given on page 485 of your textbook.

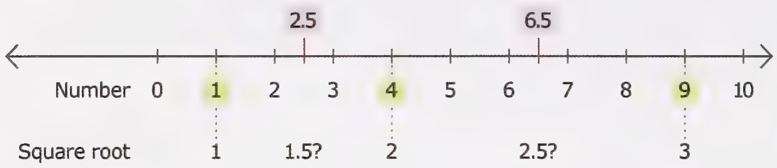
Assignment

Go to the Unit 1 Assignment Booklet, and complete "Unit 1: Lesson 4 Question Set."

Going Beyond

You estimated square roots using the square roots of perfect squares. Here is some added insight that may help you make more accurate predictions.

What if you take the square root of the exact middle number between two perfect squares? Will the answer be the exact mid-line number between the square roots of the perfect squares? For example, 1 and 4 are perfect squares. The exact middle number between them is 2.5, as shown in the illustration below.



Record your answers to the following questions to keep track of what you find.

1. Will the square root of 2.5 be 1.5, the mid-line number of the square roots of the square numbers 1 and 4? What do you think? Record your estimation, and then record what your calculator says.

2. What about the middle number of the perfect squares 4 and 9? Will the square root of 6.5 be 2.5, the mid-line number of the square roots of the square numbers 4 and 9? What do you think? Record your estimation, and then record what your calculator says.

3. What about the middle number of the perfect squares 9 and 16? Will the square root of 12.5 be 3.5, the mid-line number of the square roots of the square numbers 9 and 16? What do you think? Record your estimation, and then record what your calculator says.

4. Are the square roots getting closer to the middle number or farther away? Can you see a trend? What is it?

5. Try a couple of larger perfect squares to see if the trend holds. What do you find?

6. How large a number would you need for the square root of the middle number to be the exact mid-line number between the square roots of the perfect squares? Pick two large perfect squares, and test it out. What do you conclude?

7. What general rule would you give for estimating the square roots of the mid-line number between consecutive perfect squares?

Compare your answers in the Appendix.

Lesson Summary

In this lesson you estimated the approximate square root of numbers. Only perfect squares have square roots, which are whole numbers. In the real world, most measurements are not perfect squares, so their square roots are often expressed only as approximations. For example,

$$\sqrt{15} \approx 3.873$$

You also used a calculator or a computer to solve problems involving square roots and found that many of the answers were approximations.

You found out that you could estimate the square root of a given number by finding the closest perfect square number less than the given number and the closest perfect square number larger than the given number. You would then find the square roots of these perfect square numbers, which would be whole numbers. The square root of the given number would be between the whole numbers you found.

You were able to identify a number whose square root is between two given numbers by determining the squares of the given numbers.

Unit 1: Square Roots and the Pythagorean Theorem

Lesson 5: Introducing the Pythagorean Theorem

Get Focused

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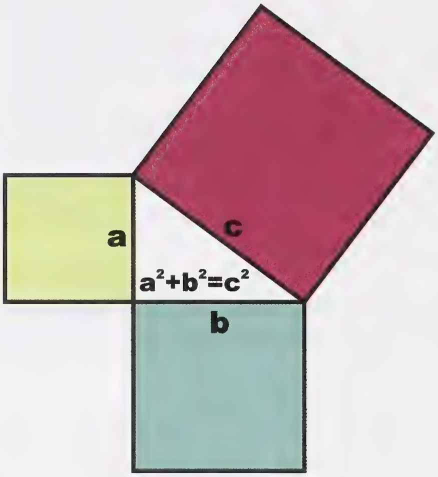

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In the last lesson, you looked at the relationship between a square's area and its square root. Building on this information, you will now see if a mathematical relationship exists between a square's area and a skateboard ramp shaped like a right-angle triangle.

For some people, a favourite pastime is skateboarding. It would be boring if all you ever did was ride along a flat surface. When dips happen, the ride becomes more exciting. As a result, developers have created skateboard parks with several different triangular shapes to ride down. This does not randomly occur. Someone purposefully created those triangles for skateboarders to enjoy.

Watch and Listen

Go to the Math 8 Multimedia DVD, and watch the video *Square Links* to see what the mathematics of triangles and square numbers has to do with skateboard parks. After you have watched the video, try "Square Roots Segment" on the Math 8 Multimedia DVD.

In this lesson you will model and explain the **Pythagorean theorem**. You will also use the Pythagorean theorem to determine whether given triangles are right triangles.

Pythagorean theorem: the sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse

Mathematically, $a^2 + b^2 = c^2$, where a, b, and c are the side lengths of the triangle and c is the hypotenuse.

This lesson will help you answer the following critical question: How can you use the Pythagorean theorem to investigate triangular structures?

You will need a calculator, protractor, scissors, transparent tape, and 1-cm grid paper for this lesson. You can find "1-cm grid paper" on the Math 8 Multimedia DVD.

Assignments

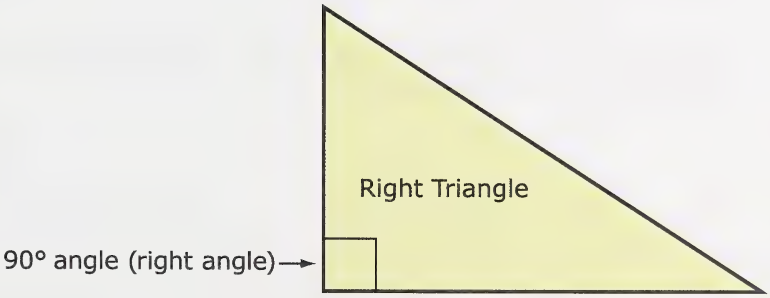
Your assignment will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 1: Lesson 5 Question Set

Explore

right triangle: a triangle that has a right angle (90° angle)

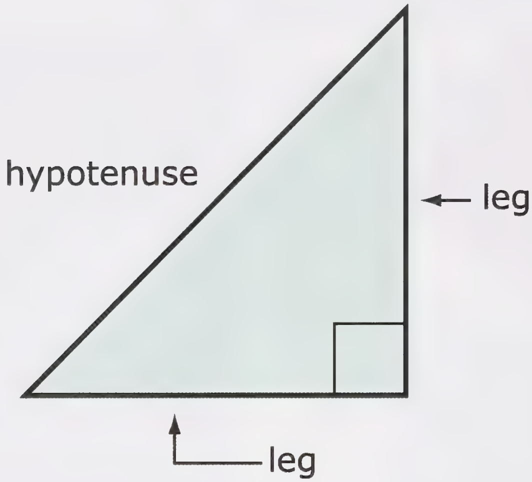
What is a **right triangle**? A right triangle is a triangle with a right angle in it. In other words, it has a 90° corner. Often, the right angle will be indicated with a small square. Right triangles are a popular shape in construction, art, sails on sailboats, and headscarves, for example.



The longest side of a right triangle is called the **hypotenuse**. It is always the side opposite the right angle. The other two shorter sides, which form the right angle, are called legs.

hypotenuse: the side of a right triangle opposite the right angle

The hypotenuse will always be the longest side of the triangle.



Try This

TT 1. Pythagoras and his theorem have been mentioned several times, but what is the theorem? Find out by doing the "Explore the Math" questions 1, 2, 3, 4, 5, 6, and 7 on pages 88 and 89 of your textbook.

Working with a partner in this activity may be beneficial, if one is available. Ask your teacher about the possibility of doing so.

Notice that for Triangle 3 in the table on page 89, you must use skills you developed in previous lessons to calculate two of the side lengths of the triangle.

My Guide

Q: Why isn't Triangle 2 a right triangle?

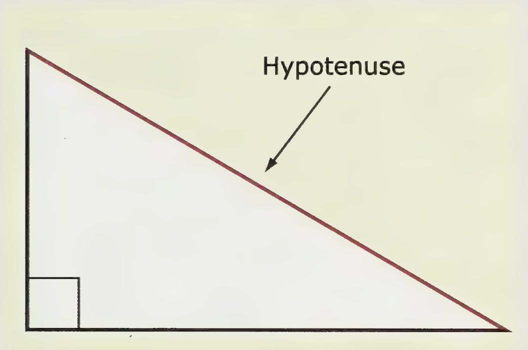
A: None of the angles in Triangle 2 are 90°.

Q: In each of the right triangles, how does the sum of the areas of the two smaller squares compare to the area of the larger square?

A: The sum of the areas of the two smaller squares is equal to the area of the larger square. (This is the Pythagorean relationship.)

Q: How can you tell which of the three sides of a right triangle is the hypotenuse?

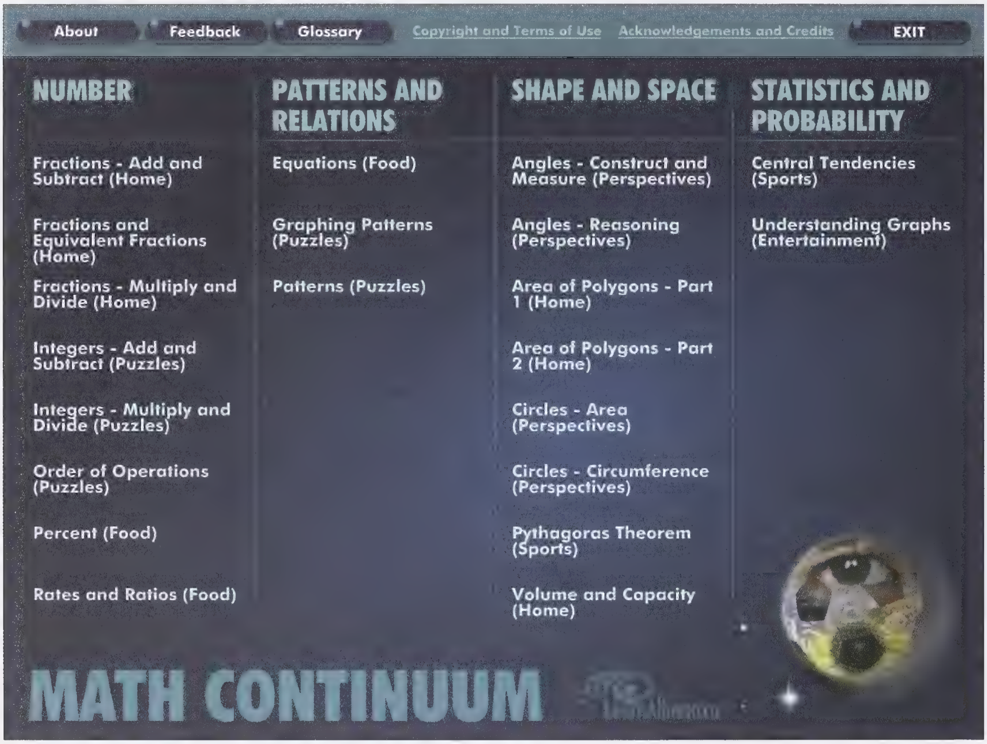
A: The hypotenuse is the longest side. It is also the side across from the right angle.

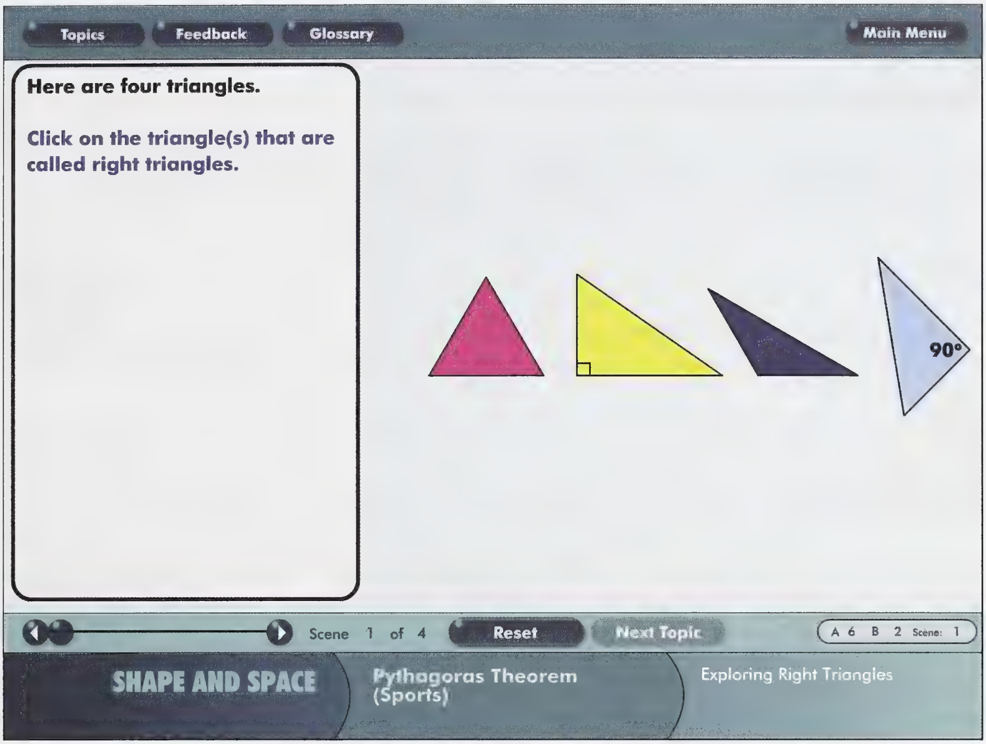


Watch and Listen

To explore the **Pythagorean relationship** further, go to the Math 8 Multimedia DVD and choose "Math Continuum." Under the heading "Shape and Space," select "Pythagoras Theorem (Sports)." Follow the directions, and answer the questions. If you make a mistake, you will be prompted so you can get the correct answer. When one topic is finished, go on to the next topic until you reach the end of "Formalizing the Pythagorean Theorem." Stop at the point where the right-hand side of the screen shows A 6 B 5 Scene 1. You will have the opportunity to go on in this program in future lessons.

Pythagorean relationship: another name for the Pythagorean theorem





Try This

TT 2. In the Pythagorean Theorem $a^2 + b^2 = c^2$, which side must always be c?

TT 3. Does the Pythagorean theorem work for triangles other than right triangles?

TT 4. Can we add the side lengths without squaring them first?

TT 5. How would you state the Pythagorean relationship in your own words?


Place a copy of your answers in your Math 8 course folder.

Discuss and Share

Post your responses to TT 2, TT 3, TT 4, and TT 5 on the discussion board. Then respond to at least two other postings.

Connect

You have just completed several topics in an interactive instructional program on the Pythagorean theorem. Sometimes it is also helpful to see it all laid out on paper.

Read

"Example 1: Describe Relationships in Right Triangles" on page 90 of your textbook allows you to see the Pythagorean theorem worked with different variables so you can identify the hypotenuse without having it labelled for you as c.

"Example 2: Identify a Right Triangle" on page 90 of your textbook shows how to tell if a triangle is a right triangle using the Pythagorean theorem. See how square numbers you have worked with in Lessons 1 through 4 are crucial in the relationships of sides in right triangles.

After you have gone over these examples, see how "Key Ideas" on page 91 condenses it all down. Note how they use different variables again for the relationship.

Now would be an excellent time to update your foldable, if you have one. If you haven't started one, perhaps now is the time to do so. Directions are given on page 78, and you can download "1-cm grid paper" from the Toolkit.

Self-Check

SC 1. Write your answers to "Check Your Understanding" questions 7, 9, and 17 on pages 92 and 94 of your textbook.

SC 2. Do "Check Your Understanding" questions 5, 8, 12, and 13 on pages 92 and 93 of your textbook.

Compare your answers in the Appendix.

Extra Practice

If you feel you have a solid understanding of how to determine the sides of a right triangle and tell whether a triangle is a right triangle, go on to the assignment. If you feel you need a bit more practise, turn to pages 92 and 93 in your textbook. Complete as many of questions 4, 6, 10, 14, and 16 from "Check Your Understanding" as you feel you need. Then check your work using the shortened answers given on pages 484 and 485 at the back of your textbook.

Assignment

To inform your teacher of your progress, this question set will need to be completed and submitted to your teacher.

Go to the Unit 1 Assignment Booklet, and complete "Unit 1: Lesson 5 Question Set."

Going Beyond

Suppose Pythagoras had worked with circles and semi-circles instead of squares. How would his theorem have been stated? Complete question 20 on page 94 of your textbook. Can you prove mathematically that this relationship would be true for the triangle shown?

Compare your answers in the Appendix.

Lesson Summary

In this lesson you used cut-out squares and formed triangles to show how the Pythagorean theorem works. You became familiar with the Pythagorean theorem stated in words and mathematical symbols. It is related to skateboard ramps because most of them are right triangles. If you know the length of any two sides, you can now determine the length of the other side.

The Pythagorean theorem states that the sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse. In mathematical symbols, it is $a^2 + b^2 = c^2$.

You were able to use the areas of squares on the sides of different triangles to demonstrate that the theorem of Pythagoras only works for right triangles.

You found that you could determine whether a triangle was a right triangle by applying the Pythagorean theorem.

Unit 1: Square Roots and the Pythagorean Theorem

Lesson 6: Exploring the Pythagorean Theorem

Get Focused

High Level Bridge at Lethbridge, Alberta, is the longest viaduct (train crossing) of its kind in the world. It was completed in 1909.

The High Level Bridge across the Oldman River valley at Lethbridge, Alberta, has carried heavy trains for over 100 years. Year after year, it has withstood terrific winds, floods, blizzards, hail, extreme cold, and blazing heat. It continues to serve well and will for a long time to come. One of the secrets of its strength is the use of triangles in its construction—the same as triangles give strength to homes in your neighbourhood.

A side view of the bridge shows the use of many right triangles in its construction. The hypotenuse of each of the crossed triangles is tied in the middle for added stability. The engineers who designed and built this structure used the Pythagorean theorem to ensure the lengths of the cross braces were exactly correct.

This is a side view of a train crossing the famous Lethbridge High Level Bridge.

In this lesson you will use the Pythagorean theorem to solve problems involving right triangles and Pythagorean triples.

This lesson will help you answer the following critical question: How can you use the Pythagorean theorem to find the length of a line segment that is part of a right triangle?

You will need 1-cm grid paper, a calculator, and a ruler for this lesson. You can find "1-cm grid paper" on the Math 8 Multimedia DVD.

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 1: Lesson 6 Question Set

Explore

Try This

TT 1. Can you find the side length of a right triangle using the Pythagorean theorem? Give it a try by doing "Explore the Math" questions 1, 2, and 3 on page 101 of your textbook.

Working with a partner in this activity may be beneficial, if one is available. Ask your teacher about the possibility of doing so.

My Guide

Q: In question 1 you were asked to draw a right triangle. Suppose the legs of the right triangle that you drew measured 4 cm and 7 cm, respectively. What is the area of the largest square—the one attached to the hypotenuse— as determined by the Pythagorean relationship?

A:
$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 4^2 + 7^2 \\ c^2 &= 16 + 49 \\ c^2 &= 65 \text{ cm}^2 \end{aligned}$$

Q: How do you determine the length of the hypotenuse to the nearest centimetre?

A: Take the square root of the area of the larger square.

$$\begin{aligned} c^2 &= 65 \\ \sqrt{c^2} &= \sqrt{65} \\ c &= \sqrt{65} \\ c &\approx 8 \text{ cm} \end{aligned}$$

Q: What are two methods you used to find the length of the hypotenuse?

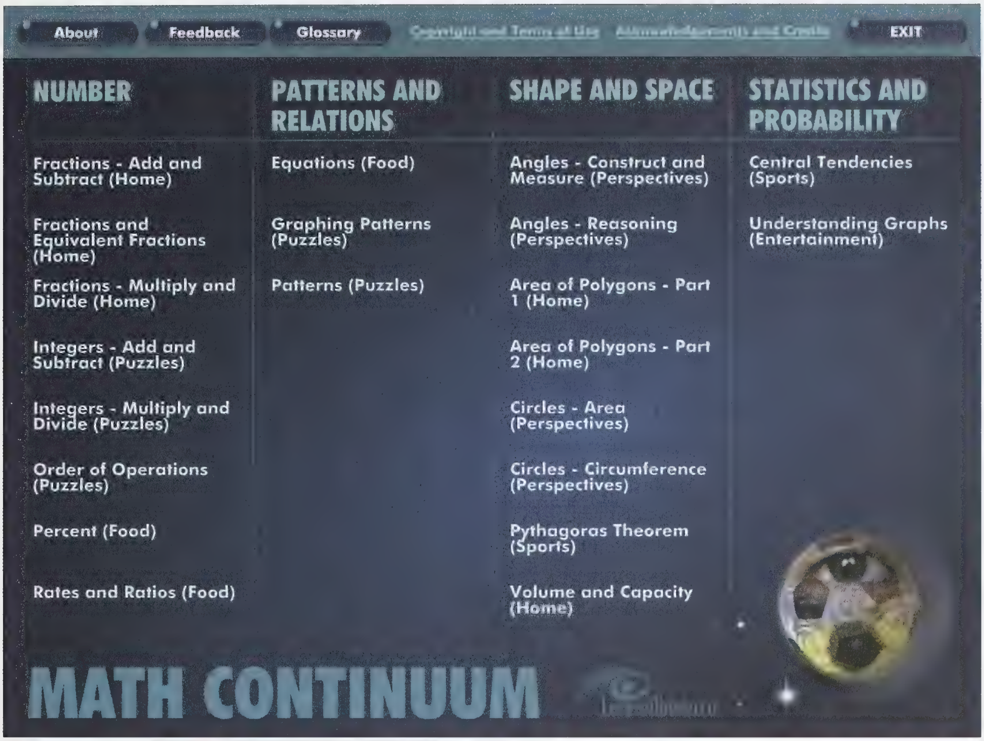
A: The hypotenuse was found using the theorem of Pythagoras and by direct measurement with a ruler.

Q: When would it be difficult to use measurement to find the length of the hypotenuse?

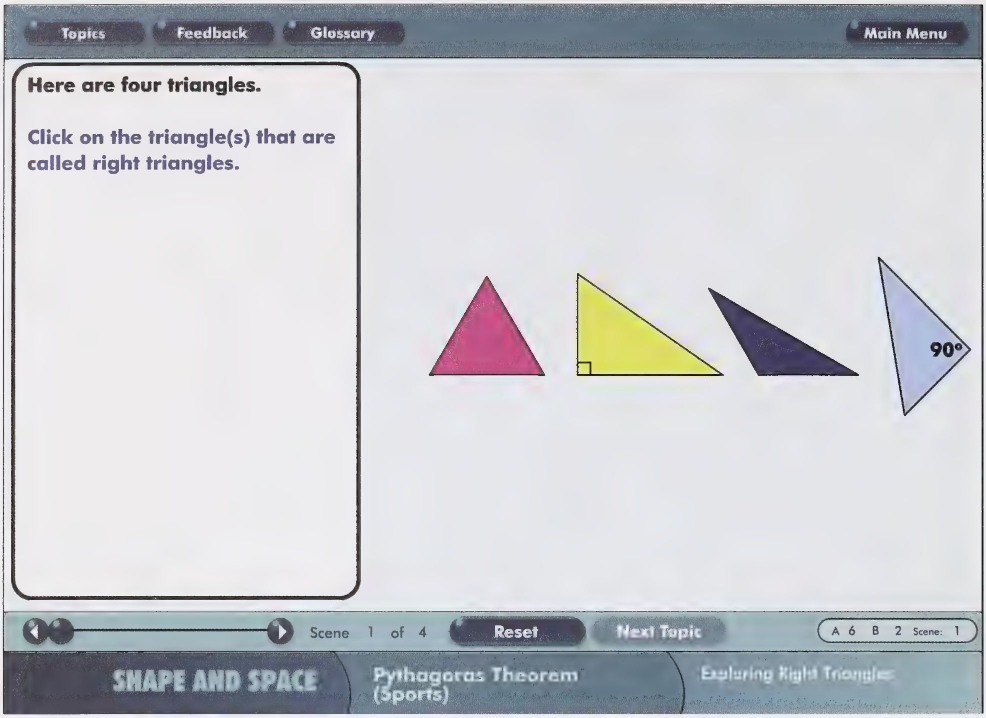
A: The hypotenuse would be difficult to find by direct measurement if it was very large or very small.

Watch and Listen

To use the Pythagorean relationship to find the length of sides of right triangle, go to the Math 8 Multimedia DVD and click on the "Math Continuum." Under the heading "Shape and Space," select "Pythagoras Theorem (Sports)."


About
Feedback
Glossary
EXIT
NUMBER
PATTERNS AND RELATIONS
SHAPE AND SPACE
STATISTICS AND PROBABILITY
Fractions - Add and Subtract (Home)
Fractions and Equivalent Fractions (Home)
Fractions - Multiply and Divide (Home)
Integers - Add and Subtract (Puzzles)
Integers - Multiply and Divide (Puzzles)
Order of Operations (Puzzles)
Percent (Food)
Rates and Ratios (Food)
Equations (Food)
Graphing Patterns (Puzzles)
Patterns (Puzzles)
Angles - Construct and Measure (Perspectives)
Angles - Reasoning (Perspectives)
Area of Polygons - Part 1 (Home)
Area of Polygons - Part 2 (Home)
Circles - Area (Perspectives)
Circles - Circumference (Perspectives)
Pythagoras Theorem (Sports)
Volume and Capacity (Home)
Central Tendencies (Sports)
Understanding Graphs (Entertainment)
MATH CONTINUUM

In the upper left-hand corner of the screen which pops up, click on "Topics."



Then select "Calculating the Measure of a Side of Right Triangles."

Follow the directions, and answer the questions. If you make a mistake, you will be prompted so you can correct your work. When you complete one topic, go on to the next topic.

Try This

TT 2. How would you state the Pythagorean relationship in the form of an equation?

TT 3. Once you have found the area of the unknown square using the Pythagorean theorem, how do you find the length of the unknown side?

TT 4. When do you add to find the area of the unknown side, and when do you subtract?

Place a copy of your answers in your Math 8 course folder.

Discuss and Share

Post your responses to TT 2, TT 3, and TT 4 on the discussion board. Then respond to at least two other postings.

Read

In the applet you were asked to find the length of the hypotenuse for a right triangle and the length of a leg for a different right triangle. To review the key differences in the approach for each case, read "Example 1: Determine the Length of the Hypotenuse of a Right Triangle," "Example 2: Determine the Length of a Leg of a Right Triangle," and "Key Ideas" on pages 102 and 103 of your textbook.

You may decide to update your foldable after you read "Key Ideas."

Self-Check

SC 1. To ensure that you know how to respond to each type of problem, whether you are asked for the length of a leg or the length of the hypotenuse, write your answers to "Practise" questions 3 and 6 on page 104 of your textbook.

Compare your answers in the Appendix.

Connect

Designing train bridges and houses requires the ability to calculate the side lengths of right triangles. Sometimes the triangles are rotated at different angles or have one leg much longer than the other. Learn to recognize the pattern and calculate the length of the third side of the right triangles in a variety of situations by completing the following Self-Check question.

Self-Check

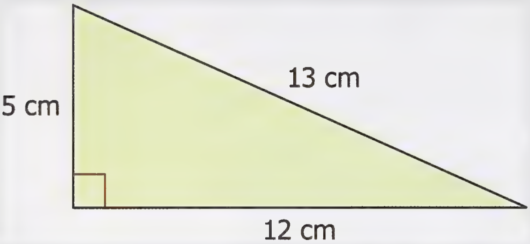
SC 2. Do "Check Your Understanding" questions 4, 7, 8, 10, and 13 on pages 104 and 105 of your textbook.

Compare your answers in the Appendix.

You may have noticed that sometimes the length of the hypotenuse turns out to be a whole number, and sometimes it turns out to be an approximation that must be rounded. When the legs of the right triangle are whole numbers and the hypotenuse is a whole number, these three numbers are called a **Pythagorean triple**. In SC 1, you completed questions 3 and 6 on page 94 of your textbook that involved Pythagorean triples. However, questions 4 and 7 of SC 2 did not, because you had to round the answer to the nearest tenth. Look back at your answers to these questions to confirm the difference.

Pythagorean triple: three whole numbers that form the sides of a right triangle

Example: 5, 12, 13



Self-Check

SC 3. Work with one of the Pythagorean triples as you complete "Extend" question 21 on page 94 of your textbook.

Compare your answers in the Appendix.

Extra Practice

If you feel you have a solid understanding of how to determine the sides of any right triangle and tell whether a Pythagorean triple is involved, go on to the assignment. If you feel you need a bit more practise, you may complete questions 5, 9, 11, 12, and 14 from "Check Your Understanding" on pages 104 and 105 in your textbook. Notice how question 5 involves a Pythagorean triple. When you finish each question, check your work using the shortened answers given on page 485 at the back of your textbook.

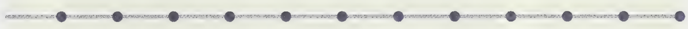
Assignment

To inform your teacher of your progress, this question set will need to be completed and submitted to your teacher.

Go to the Unit 1 Assignment Booklet, and complete "Unit 1: Lesson 6 Question Set."

Going Beyond

An old Boy Scout manual gave directions for indirectly measuring the distance straight across a stream that involved a Pythagorean triple. Part of the procedure used a rope as a measuring device, with 12 knots tied exactly the same distance apart, starting that same distance from the end as shown below. (The Scout manual was written some time ago, so the knots were to be placed 1 foot apart or about 30.5 cm.)



In the procedure, the rope had to be arranged into a right triangle, but there was no protractor or square to ensure the angle was correct. How would you arrange the rope to form an exact right triangle as the procedure required? Try drawing different arrangements of the rope in triangles with knots at each vertex, till you are sure it is a right triangle.

Compare your answers in the Appendix.

Lesson Summary

Engineers who design train bridges, like High Level Bridge at Lethbridge, use the Pythagorean theorem to calculate the side lengths of right triangles. In the same way, in this lesson you determined the side lengths of various right triangles. You found the length of the hypotenuse for some triangles by adding the squares of the lengths of the two legs and then taking the square root. As well, you found the length of a leg in other triangles by finding the difference between the squares of the other two sides and then taking the square root.

You also discovered that for some right triangles, the lengths of the sides only involve whole numbers and are called Pythagorean triples.

Unit 1: Square Roots and the Pythagorean Theorem

Lesson 7: Applying the Pythagorean Theorem

Get Focused

Whether it is new construction of a house or a stairway that has seen some time, right triangles and the Pythagorean theorem define relationships of distances. Sometimes, distances can't be determined with a carpenter's tape, either, because obstacles are in the way or the distances are too large. In these cases, the Pythagorean theorem has often been of assistance.

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In this lesson you will apply the Pythagorean theorem to solve problems of a practical nature involving right triangles, some involving long distances.

This lesson will help you answer the following critical question: How can you use the Pythagorean theorem to find the length of diagonals in structures and to solve other problems such as distances on Earth's surface?

You will need a calculator for this lesson.

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 1: Lesson 7 Question Set

Explore

In the last lesson you used the Pythagorean relationship to determine the side lengths of right angle triangles that were measured in centimetres and metres. What happens when one triangle side measurement is missing and the distances involve hundreds of metres or kilometres?

Read

Turn to page 106 of your textbook, and read up to “Explore the Math” to see a situation where the Pythagorean relationship can be used to find the side length of a very large right triangle. Does a strategy come to mind to answer the question? You will soon have an opportunity to try it.

Try This

TT 1. Use the Pythagorean relationship to measure distances that would otherwise be very difficult to determine by doing “Explore the Math” questions 1, 2, and 3 on page 106 of your textbook. Working with a partner in this activity may be beneficial, if one is available. Ask your teacher about the possibility of doing so.

My Guide

Q: What is the distance the crow flies from the house to the school (i.e., the hypotenuse) as determined by the Pythagorean relationship?

A:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 400^2 + 600^2 \\ c^2 &= 160\ 000 + 360\ 000 \\ c^2 &= 520\ 000 \\ \sqrt{c^2} &= \sqrt{520\ 000} \\ c &\approx 721.1 \end{aligned}$$

The crow flies approximately 721.1 m.

Q: What is the distance that Sam walks?

A: Sam walks 400 m + 600 m = 1000 m.

Q: How much farther does Sam travel?

A: Sam walks 1000 m − 721.1 m = 278.9 m.

Q: Why is the distance that the crow flies difficult to measure?

A: Between Sam's house and the school, one could encounter houses, fences, stores, and office buildings that would interfere with the measuring process.

TT 2. Answer the "Show You Know" question on the bottom of page 107 in your textbook.

TT 3. How did you know whether to add or subtract?

TT 4. In what ways could the ship's navigator check that his calculations for distance were correct that he couldn't do if he lived 70 years ago?

Discuss and Share

Post your responses to TT 2, TT 3, and TT 4 on the discussion board. Then respond to at least two other postings.

Connect

Read

Read "Example 1: Determine Distances With Right Triangles" on page 107 of your textbook, and then read "Example 2: Verify a Right Angle Triangle" on page 108. Look particularly at how the Pythagorean relationship is used to solve different types of practical problems. In Example 2, the "Left Side" and "Right Side" refer to sides of the Pythagorean theorem equation.

$$a^2 + b^2 = c^2$$

Left Side, Right Side

Self-Check

SC 1. Answer the "Show You Know" question on the bottom of page 108 of your textbook.

SC 2. Answer "Practise" questions 3 and 6 on page 110 of your textbook.

Compare your answers in the Appendix.

Now may be a good time to update your foldable, if you have added insights to record.

Self-Check

SC 3. Do "Check Your Understanding" questions 4, 5, 9, and 10 on page 110 of your textbook. Make diagrams of the situations involved, if that helps you understand the problem better.

Compare your answers in the Appendix.

Extra Practice

If you feel you have a solid understanding of how to determine the sides of a right triangle and how to tell whether a triangle is a right triangle, go on to the assignment. If you feel you need a bit more practise, do "Check Your Understanding" questions 7, 8, and 11 on pages 110 and 111 of your textbook. Make diagrams of the situations involved, if that helps you understand the problem better. When you have finished a question, check your work using the answers given on page 485 at the back of your textbook.

Assignment

To inform your teacher of your progress, this question set will need to be completed and submitted to your teacher.

Go to the Unit 1 Assignment Booklet, and complete "Unit 1: Lesson 7 Question Set."

Going Beyond

Sometimes more than one calculation is needed to arrive at an answer. Look at "Extend" question 16 on page 105 of your textbook. The dotted red diagonal of the tiny box is the hypotenuse of a right triangle, but you can't use the Pythagorean relationship at first to calculate the length of the red diagonal because you don't know the length of the bottom leg. The vertical leg of the right triangle is 5 mm, which you get from the height of the box.

However, the bottom leg is a diagonal of the bottom of the box. You can use the Pythagorean relationship to calculate the diagonal of the bottom of the box, since it is the hypotenuse of a right triangle with legs 12 mm and 7 mm in length.

Calculate the length of the diagonal of the bottom of the box. Then you can use that value to find the red diagonal of the tiny box.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you applied the Pythagorean theorem to solve practical problems in construction situations and in measuring distances. The diagonal of a rectangle is the hypotenuse of a right triangle. You have also used this fact to determine if an angle is a right angle. You can also use this fact to measure the length of the diagonal in several rectangles.

You were able to calculate distances that would be very difficult to measure directly. Some distances were diagonals of rectangles, and some were legs of right triangles. In both cases, the Pythagorean theorem was the key to finding the answer.

Unit 1: Square Roots and the Pythagorean Theorem

Unit 1 Summary

Getting Started

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When a carpenter is building a set of stairs in a house, the boards that form the sides of the staircase and support the steps are called stringers. In the picture above, a carpenter is marking out the cuts to be made in a stringer. You will be asked to calculate the length of a couple of stringers as you complete the Unit Problem for Unit 1. You can do this calculation because a stringer forms the hypotenuse of a right triangle.

The Unit Problem is the "Challenge in Real Life" on page 117 of your textbook. In this unit project you will use the theorem of Pythagoras to calculate the length of a stringer and also to design a step stool with a stringer. Ask your teacher how your work will be evaluated on this project, if you have not already been informed.

Read

To see exactly what is required in this project, read page 117 in your textbook. For question 4, if you want 1-cm grid paper on which to draw your design, you can access it in the Toolkit.

Try This

TT 1. To demonstrate to your teacher some of the skills and abilities you have learned in Unit 1, answer questions 1, 2, 3, and 4 on page 117 completely. When you are satisfied with your answers, submit them to your teacher. If you need help getting started with the first couple of questions, consult My Guide for direction.

My Guide

Q: How do you calculate the total rise of the staircase?

A: To calculate the total rise of the staircase, multiply the number of steps by the rise of each step. The answer will be the total rise in centimetres.

Q: How do you calculate the total run of the staircase?

A: To calculate the total run of the staircase, multiply the number of steps by the run of each step. The answer will be the total run in centimetres.

Q: What are the legs of the right triangle in this situation?

A: The legs of the right triangle are the total rise of the staircase and the total run of the staircase.

Unit Summary

In this unit you used square tiles and grid paper to model the relationship between the side lengths of squares and their areas. You used those concepts to gain an understanding of the terms square number (perfect square) and square root. You practised estimating square roots for numbers that were not perfect squares by using both the square roots of perfect squares for comparison and a calculator or computer.

When you applied your knowledge of squares and square roots to the sides of right triangles, you were led to discover the Pythagorean relationship: The sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse. This relationship modelled on grid paper was expressed mathematically as $a^2 + b^2 = c^2$, where a and b are the lengths of the triangle sides attached at right angles and c is the hypotenuse (remember it is opposite the right angle).

You were able to use the Pythagorean relationship to calculate the side lengths of various other right triangles. You also used the mathematical formula to determine whether or not a triangle was a right triangle. You were able to use the Pythagorean relationship in construction situations and in determining measurement of both great length and very tiny dimensions.

This unit helped you answer the following critical question: How are properties of square numbers and the Pythagorean Theorem used in identifying patterns, measuring lengths, and designing structures?

Unit Review

In order to reinforce the concepts and skills of this unit, look over your foldable until all the ideas are fresh in your mind. Then complete some questions that prompt you think back on what you have learned.

Self-Check

SC 1. Turn to “Chapter 3 Review” on pages 112 and 113. Complete questions 1, 2, 3, 4, and 5 and at least two questions from each of the five text sections shown.

Compare your answers in the Appendix.

Are You Ready?

You've done a lot of work to reach this point in the unit, haven't you? But are you ready to strut your stuff—that is, show your mastery of the new concepts and skills? Are you really ready to take the challenge of a unit test? You may feel you are ready but you'd like to do a practice test before doing one for marks. Well, here's your chance. Complete the following Self-Check question.

Self-Check

SC 2. Turn to pages 114 and 115 and complete "Practice 3 Test." You may use a calculator or the computer program you developed to calculate squares and square roots.

Compare your answers in the Appendix.

When you are finished, submit the answers to your teacher to correct. Your mark will not be used for report cards. It is just to confirm in your own mind how well you can do at this point. Then if you have some questions, you can get them answered ahead of time. Or, if you feel you might need some more support in an area, you can ask for a little help.

Assignment

Check with your teacher about a unit test.

Appendix

Lesson 1

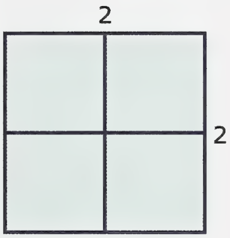
SC 1. Yes. A square is a quadrilateral that has four right angles. That fits the definition of a rectangle.

SC 2.

5. a. $4 = 2 \times 2$

   b. Yes, 4 is a perfect square because the prime factor 2 appears an even number of times.

   c.



7. a. $42 = 2 \times 3 \times 7$

   Therefore, 42 is not a perfect square.

   b. $169 = 13 \times 13$

   Therefore, 169 is a perfect square.

   c. $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

   Therefore, 256 is a perfect square.

17. $54 = 2 \times 3 \times 3 \times 3$

    No, 54 is not a perfect square because it has an odd number of both the factors 2 and 3.

21. a. 56 m^2

    b. Answers may vary. One possibility is 7 m by 8 m, and another is 2 m by 28 m.

    c. No, it is not possible because 56 is not a perfect square.

23. $400 = (2 \times 2 \times 5) \times (2 \times 2 \times 5)$

    The side length is 20 m.

Going Beyond

24.
    a. The next three triangular numbers are 10, 15, and 21.

    b. When you add together any two consecutive triangular numbers, the sum is always a perfect square. For example,

       $1 + 3 = 4$
       $3 + 6 = 9$
       $6 + 10 = 16$
       $10 + 15 = 25$

Lesson 2

SC 1. The factors of 10 are 1, 2, 5, and 10.

SC 2. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

SC 3.

9.
    a. 100 square units

    b. 256 square units

11.
    a. 81

    b. 121

13. 7 mm

14. 30 cm

15.
    a. 7

    b. 8

    c. 25

18. 196 m^2

Going Beyond

27.

a. $\sqrt{6400} = 80$, $\sqrt{640\ 000} = 800$, $\sqrt{64\ 000\ 000} = 8000$

b. Take the square root of 64 and add half the number of zeros following 64 in the original number.

c. The number of trailing zeros is not an even number.

d. $\sqrt{640\ 000\ 000\ 000} = 800\ 000$

Write the square root of 64, which is 8, and then add half the number of trailing zeros ($10 \div 2 = 5$) in the original number to the 8.

Lesson 3

SC 1.

19. Let the side length of the square in metres be represented by the italicized lower case letter L (shown as *l*).

$$\begin{aligned} l &= \sqrt{28900} \\ &= 170 \end{aligned}$$

The perimeter is 4 *l* and the students run twice around, so the total distance is 8 *l*.

8×170 m = 1360 m

The students ran 1360 m.

20. a. $A = l \times w$

$A = 9\ \text{m} \times 4\ \text{m}$

$A = 36\ \text{m}^2$

The are of the rectangle is 36 m^2.

b. The area of the square equals the area of the rectangle.

The side length of the square in metres is represented by *s*.

$$\begin{aligned} A &= s^2 \\ 36 &= s^2 \\ \sqrt{36} &= s \\ 6 &= s \end{aligned}$$

The side length of the square is 6 m.

22.

c. To find the dimensions in whole numbers, take the square root of both the minimum and maximum sizes. The approximate square roots are given, because neither of the numbers is a perfect square.

$$\sqrt{386\ 000} = 621.3 \text{ and } \sqrt{394\ 000} = 627.7$$

Therefore, the whole number dimensions must lie between the two numbers. The possibilities are 622 m by 622 m, 623 m by 623 m, 624 m by 624 m, 625 m by 625 m, 626 m by 626 m, and 627 m by 627 m.

SC 2.

a. Using strategy 2, the area of the enclosing square is 81 cm^2. There are four congruent triangles enclosed by one of the sides of the angled square and two sides of the enclosing square. Each of these triangles has an area of 7 cm^2.

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}\,\text{base} \times \text{height} \\ &= \tfrac{1}{2} \times 7 \text{ cm} \times 2 \text{ cm} \\ &= 7 \text{ cm}^2 \end{aligned}$$

The combined area of these triangles is 28 cm^2.

Alternately, combining opposite triangles into two rectangles, each of which has an area of 14 cm^2, gives the combined area of 28 cm^2.

The area of the angled square is equal to 81 cm^2 minus 28 cm^2, which equals 53 cm^2. The area of the orange square is 53 cm^2.

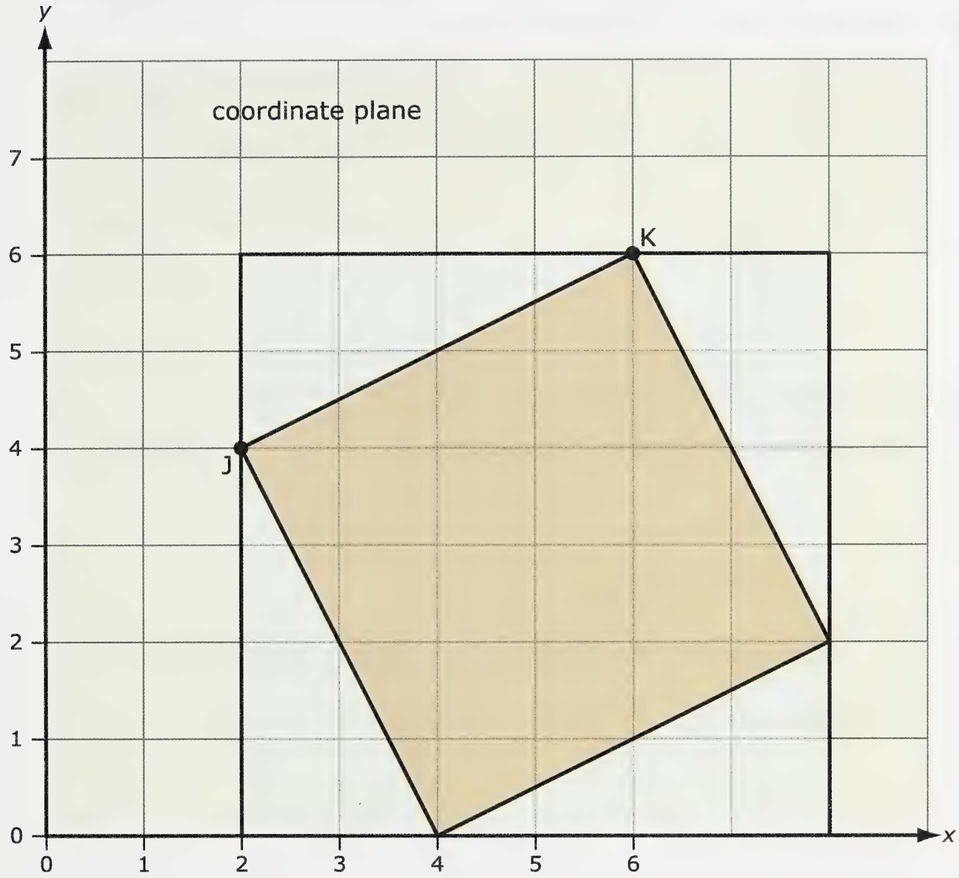
b.
$$\begin{aligned} s &= \sqrt{A} \\ &= \sqrt{53 \text{ cm}^2} \\ &\approx 7.3 \text{ cm} \end{aligned}$$

The side length of the orange square is $\sqrt{53}$ cm, which makes it approximately 7.3 cm long.

Extra Practice

The area of the tilted square is 17 cm^2, and the side length is $\sqrt{17}$ cm or approximately 4.1 cm.

Going Beyond



A tilted square having one side as $\overline{JK}$ is drawn. The area of the enclosing square is 36 square units. There are four congruent triangles outside the angled square but inside the enclosing square. Each of these triangles has an area of 4 square units. The combined area of these triangles is 16 square units. The area of the tilted square is equal to 36 square units minus 16 square units, which equals 20 square units.

Therefore, the length of the line segment $\overline{JK}$ is $\sqrt{20}$ units.

Lesson 4

SC 1. The closest perfect square number less than 28 is 25. The closest perfect square number greater than 28 is 36. So 25 and 36 are the most useful for the estimation. They show that the square root of 28 is between 5 and 6.

SC 2. The closest perfect square number less than 350 is 324. The closest perfect square number greater than 350 is 361. So 324 and 361 are the most useful for the estimation. These perfect square numbers show that $\sqrt{350}$ is between 18 and 19.

SC 3.

4. Answers may vary slightly from those given because they are estimates.

   a. 8.5

   b. 10.1

   c. 7.4

10. Answer may vary slightly because it is an estimate. Sample: 5.2 m.

12. Your answer may differ because it is an estimate.

   a. $\sqrt{11} \approx 3.3$ m

   b. $\sqrt{11} \approx 3.31662479$

   c. Yes, the rug will fit because each side is smaller than the shortest side of the room.

15. $\sqrt{27}$, 5.8, 6.3, $\sqrt{46}$, 7

SC 4.

2. Any of the whole numbers 10, 11, 12, 13, 14, and 15 would be correct. To determine the number, take the square of 3 and the square of 4, which are 9 and 16, respectively. Then choose any whole number between 9 and 16.

3. The square root of 10 displayed on the calculator is only an approximation because it is not a perfect square. When the number shown on the calculator is multiplied by itself, the answer will be approximately 10, but not exactly 10.

6. Any number between 82 and 99, inclusive, is correct. Example: 92.

8. 5, 6, 7, and 8

14.

   a. Alex could be thinking of 60.

   b. No, there is no other number between 49 and 64 that is a multiple of 12.

Going Beyond

1. $\sqrt{2.5} \approx 1.58$

   It is larger than the mid-line number.

2. $\sqrt{6.5} \approx 2.55$

   It is larger than the mid-line number.

3. $\sqrt{12.5} \approx 3.54$

   It is larger than the mid-line number.

4. The square roots are getting closer to the mid-line number as the size of the perfect squares increases.

5. The following table gives some values for the numbers already tried and for larger numbers. As the numbers get larger, the square roots get closer to the expected mid-line.

Square root	Square number	Middle number	Square root	Expected Mid-line
1	1			
		2.5	1.58113883	1.5
2	4			
		6.5	2.549509757	2.5
3	9			
		12.5	3.535533906	3.5
4	16			
		20.5	4.527692569	4.5
5	25			
		30.5	5.522680509	5.5
6	36			
		42.5	6.519202405	6.5
7	49			
		56.5	7.516648189	7.5
8	64			
		72.5	8.514693183	8.5
9	81			
		90.5	9.513148795	9.5
10	100			
		110.5	10.51189802	10.5
11	121			
100	10 000			
		10 100.5	100.5012438	100.5
101	10 201			

6. As the numbers get very large, the square root will get ever closer to the mid-line but will never be exactly equal to it.

7. Answers may vary. Here is a sample:

   For smaller numbers, the square root of the middle number between two perfect squares will be slightly larger than the mid-line number between their square roots. As the size of the number increases, the difference between them decreases.

Lesson 5

SC 1.

7.

a. $(9\text{ cm})^2 = 81\text{ cm}^2$, $(12\text{ cm})^2 = 144\text{ cm}^2$, $(15\text{ cm})^2 = 225\text{ cm}^2$

b. $81\text{ cm}^2 + 144\text{ cm}^2 = 225\text{ cm}^2$

c. The sum of squares of the legs of the triangle equals the square of the length of the longest side $(9\text{ cm})^2 + (12\text{ cm})^2 = (15\text{ cm})^2$.

9.

a. $(2\text{ cm})^2 = 4\text{ cm}^2$, $(3\text{ cm})^2 = 9\text{ cm}^2$, $(4\text{ cm})^2 = 16\text{ cm}^2$

b. No, this is not a right triangle because the sum of the squares of the shorter sides does not equal the sum of the square of the hypotenuse.

   $4\text{ cm}^2 + 9\text{ cm}^2 \neq 16\text{ cm}^2$

17.

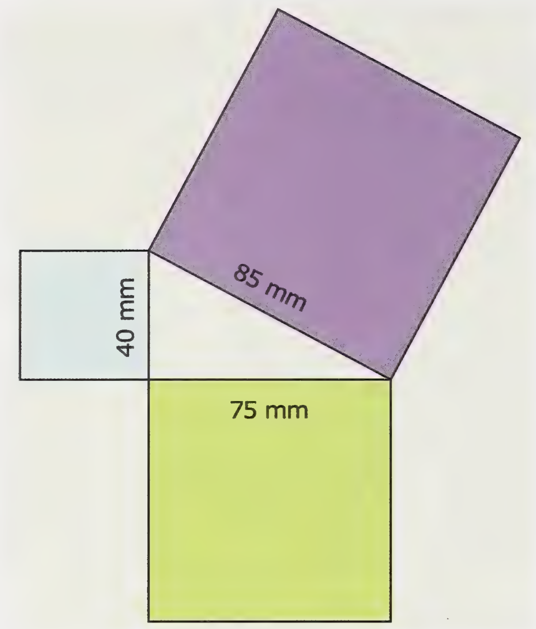
a. $c^2 = 21^2 + 28^2$
   $c^2 = 441 + 784$
   $c^2 = 1225\text{ cm}^2$

b. $c^2 = 5^2 + 12^2$
   $c^2 = 25 + 144$
   $c^2 = 169\text{ cm}^2$

SC 2.

5.

a.



b. The 40 mm sided square has an area of $(40 \text{ mm})^2 = 1600 \text{ mm}^2$.

The 75 mm sided square has an area of $(75 \text{ mm})^2 = 5625 \text{ mm}^2$.

The 85 mm sided square has an area of $(85 \text{ mm})^2 = 7225 \text{ mm}^2$.

c. $1600 \text{ mm}^2 + 5625 \text{ mm}^2 = 7225 \text{ mm}^2$

8. The shown triangle is not a right triangle. The sum of the areas of the two smaller squares are greater than the area of the largest square. Only when the sum of the areas of the smaller squares equals the area of the largest square is the triangle a right triangle.

12.

a. $20 \text{ cm}^2 + 32 \text{ cm}^2 = 52 \text{ cm}^2$, so unknown area is 52 cm^2.

b. $100 \text{ mm}^2 + 576 \text{ mm}^2 = 676 \text{ mm}^2$, so unknown area is 676 mm^2.

c. $90 \text{ cm}^2 - 25 \text{ cm}^2 = 65 \text{ cm}^2$, so unknown area is 65 cm^2.

d. $12 \text{ cm}^2 + 12 \text{ cm}^2 = 24 \text{ cm}^2$, so unknown area is 24 cm^2.

13. The shown triangle is not a right triangle. The sum of the areas of the two smaller squares is less than the area of the largest square.

    $4800 \text{ cm}^2 + 4800 \text{ cm}^2 = 9600 \text{ cm}^2$
    $9600 \text{ cm}^2 < 9800 \text{ cm}^2$

    Only when the sum of the areas of the smaller squares equals the area of the largest square is the triangle a right triangle.

Going Beyond

20. Answers may vary. Here is a sample: The sum of the areas of the semi-circles attached to the two smaller sides of a right triangle is equal to the area of the semi-circle attached to the hypotenuse.

    To check mathematically:

    The area of a semi-circle is $\frac{1}{2}\pi r^2$, where r is half the side length of the triangle.

    $$\frac{1}{2}\pi ra^2 + \frac{1}{2}\pi rb^2 = \frac{1}{2}\pi rc^2$$

    We could insert the numbers into the equation and calculate now, but it will be easier if we simplify it first.

    Dividing each term in the equation by π and multiplying by 4 we get

    $$\begin{aligned} 2ra^2 + 2rb^2 &= 2rc^2 \\ 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \\ 25 &= 25 \end{aligned}$$

    Yes, the theorem would work if stated in semi-circles instead of squares.

Lesson 6

SC 1.

3.

   a. $$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 12^2 + 16^2 \\ c^2 &= 144 + 256 \\ c^2 &= 400 \\ \sqrt{c^2} &= \sqrt{400} \\ c &= 20 \text{ cm} \end{aligned}$$

b. $r^2 = p^2 + q^2$

$r^2 = 16^2 + 30^2$

$r^2 = 256 + 900$

$r^2 = 1156$

$\sqrt{r^2} = \sqrt{1156}$

$r^2 = 34$ m

6.

a. $a^2 + b^2 = c^2$

$7^2 + b^2 = 25^2$

$49 + b^2 = 625$

$b^2 = 625 - 49$

$c^2 = 576$

$\sqrt{c^2} = \sqrt{576}$

$c = 24$ cm

b. $r^2 + s^2 = t^2$

$r^2 + 24^2 = 26^2$

$r^2 + 576 = 676$

$r^2 = 676 - 576$

$r^2 = 100$

$\sqrt{r^2} = \sqrt{100}$

$r = 10$ cm

SC 2.

4.

a. $(7\text{ cm})^2 + (6\text{ cm})^2 = (49 + 36)\text{cm}^2 = 85\text{ cm}^2$ $\quad \sqrt{85\text{ cm}^2} \approx 9.2\text{ cm}$

b. $(11\text{ cm})^2 + (8\text{ cm})^2 = (121 + 64)\text{cm}^2 = 185\text{ cm}^2$ $\quad \sqrt{185\text{ cm}^2} \approx 13.6\text{ cm}$

7.

a. $(9\text{ mm})^2 - (5\text{ mm})^2 = (81 - 25)\text{mm}^2 = 56\text{ mm}^2$ $\quad \sqrt{56\text{ mm}^2} \approx 7.5\text{ mm}$

b. $(15\text{ mm})^2 + (11\text{ mm})^2 = (225 + 121)\text{mm}^2$
$= 104\text{ mm}^2$ $\quad \sqrt{104\text{ mm}^2} \approx 10.2\text{ mm}$

8. $(200\text{ cm})^2 + (50\text{ cm})^2 = (40\ 000 + 2500)\text{cm}^2$
$= 42\ 500\text{ cm}^2$ $\quad \sqrt{42\ 500\text{ cm}^2} \approx 206.2\text{ cm}$

10. $(27\text{ m})^2 + (27\text{ m})^2 = (729 + 729)\text{m}^2$
$= 1458\text{ m}^2$ $\quad \sqrt{1458\text{ m}^2} \approx 38.2\text{ m}$

13. $(10\text{ mm})^2 - (8\text{ mm})^2 = (100 - 64)\text{mm}^2$
$= 36\text{ mm}^2$ $\quad \sqrt{36\text{ mm}^2} \approx 6.0\text{ mm}$

The length of the base of the large triangle is twice the base of the smaller triangles, or 12 mm.

SC 3.

21.

a.
$$\begin{aligned}(2\times3)^2 + (2\times4)^2 &= (2\times5)^2\\ 6^2 + 8^2 &= 10^2\\ 36 + 64 &= 100\\ 100 &= 100\end{aligned}$$

b.
$$\begin{aligned}(5\times3)^2 + (5\times4)^2 &\doteq (5\times5)^2\\ 15^2 + 20^2 &= 25^2\\ 225 + 400 &= 625\\ 625 &= 625\end{aligned}$$

c.
$$(3\times x)^2 + (4\times x)^2 = (5\times x)^2$$
$$9x^2 + 16x^2 = 25x^2$$
$$25x^2 = 25x^2$$

No matter which natural number you use, you will end with a Pythagorean triple.

Going Beyond

The rope must be arranged so knots form vertices of a right triangle with side lengths of 3 units, 4 units, and 5 units. That takes exactly 12 knots of rope. The numbers 3, 4, and 5 form a Pythagorean triple. You may remember solving several questions in this unit with triangles whose side lengths were these numbers.

Lesson 7

SC 1.

Left Side:

$a^2 + b^2 = 17^2 + 20^2$
$= 289 + 400 = 689$

The sum of the areas of the two smaller sides is 689 m^2.

Right Side:

$c^2 = 26.25^2 = 689.0625$

The area of the large square is 689.0625 m^2.

The left side and right side are very close to the same area. The very small difference could be caused by a slight approximation in any of the measurements. Therefore, the corner is a right angle.

SC 2.

3.

a. Maria walked 120 m + 300 m, so she walked 420 m.

b.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 120^2 + 300^2 \\ c^2 &= 14\ 400 + 90\ 000 \\ c^2 &= 104\ 400 \\ \sqrt{c^2} &= \sqrt{104\ 400} \\ c &= 323 \end{aligned}$$

Walter walked 323 m.

c. $420 - 323 = 97$

Maria walked 97 m farther than Walter.

6. The red lines form three sides of a triangle.

Left Side:

$a^2 + b^2 = 27^2 + 27^2$
$= 729 + 729 = 1458$

The sum of the areas of the two smaller sides is 1458 m^2.

Right Side:

$c^2 = 37.1^2 = 1376.41$

The area of the large square is 1376.41 m^2.

The areas on the left side and right side are not equal, so the triangle is not a right triangle.

SC 3.

4. The height of the pole is one side of a right triangle.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ (10\text{ m})^2 &= (2\text{ m})^2 + b^2 \\ b^2 &= (100 - 4)\text{m}^2 \\ b &= \sqrt{96\text{ m}^2} \\ b &\approx 9.8\text{ m} \end{aligned}$$

The pole is about 9.8 m tall.

5. The width, length, and diagonal of a rectangle form a right triangle.

$$\begin{aligned} (22\text{ cm})^2 + (9\text{ cm})^2 &= 484\text{ cm}^2 + 81\text{ cm}^2 \\ (23.8\text{ cm})^2 &= 566.44\text{ cm}^2 \\ 566.44\text{ cm}^2 &\approx 565\text{ cm}^2 \end{aligned}$$

These measurements could come from a rectangle.

9.

a. $$(3\text{ cm})^2 + (3\text{ cm})^2 = 9\text{ cm}^2 + 9\text{ cm}^2$$
$$\sqrt{18\text{ cm}^2} \approx 4.2\text{ cm}$$

The diagonal of a small square is about 4.2 cm.

b. $$(3 \times 8\text{ cm})^2 + (3 \times 8\text{ cm})^2 = 576\text{ cm}^2 + 576\text{ cm}^2$$
$$\sqrt{1152\text{ cm}^2} \approx 33.9\text{ cm}$$

The diagonal is about 33.9 cm long.

You could also accept 8 × 4.2 cm = 33.6 cm. Notice how doing calculations with rounded numbers leads to less accurate results.

10. $$(12\text{ m})^2 + (12\text{ m})^2 = 144\text{ m}^2 + 144\text{ m}^2$$
$$\sqrt{288\text{ m}^2} \approx 17.0\text{ m}$$

The gymnast will have room since the diagonal of the mat is about 17.0 m.

Going Beyond

16. The diagonal of the base of the box can be calculated as follows:

$$\begin{aligned}
c^2 &= a^2 + b^2 \\
c^2 &= (12\text{ mm})^2 + (7\text{ mm})^2 \\
c^2 &= 193\text{ mm}^2 \\
\sqrt{c^2} &= \sqrt{193\text{ mm}^2} \\
c &\approx 13.89244\text{ mm}
\end{aligned}$$

Now you can use this length to calculate the length of the red diagonal in the diagram.

$$\begin{aligned}
c^2 &= a^2 + b^2 \\
c^2 &= (13.89244\text{ mm})^2 + (5\text{ mm})^2 \\
c^2 &= (193 + 25)\text{mm}^2 \\
c^2 &= 218\text{ mm}^2 \\
\sqrt{c^2} &= \sqrt{218\text{ mm}^2} \\
c &\approx 14.8\text{ mm}
\end{aligned}$$

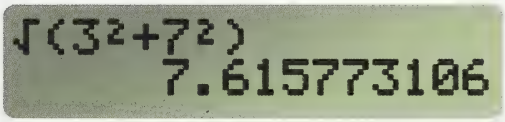
The red diagonal is about 14.8 mm long.

Unit Summary

SC 1. Check your answers after completing each section, using the solutions provided on page 486 in your textbook. You may do the additional questions if you feel you need the practise.

SC 2.

1. D $100 = 10^2$
2. B $81 = 9^2$
3. D $49 = 7^2$
4. D $(6\text{ m}^2 + 22\text{ m}^2 = 28\text{ m}^2)$
5. A $\sqrt{51} \approx 7.14$
6. c $\sqrt{53\text{ cm}^2} \approx 7.3\text{ cm}$
7. 7.1 cm
8. a. Using a calculator, your answer should look like the following:



The hypotenuse is approximately 7.6 cm.

b. The answer given by the calculator is approximate although it is quite accurate. When you round the answer, it is still approximate and has lost some accuracy.

9. The float line is the same length as the shorter side of the pool, so it is 8 m.
10. a. To have a square root between 7 and 8, the number must be between 49 and 64.

b. There are 15 whole numbers with square roots between 7 and 8. They are 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, and 63.

11. A triangle with sides of 14 mm, 48 mm, and 50 mm is a right triangle since the sides are a Pythagorean triple.

$$\begin{aligned} 50^2 &= 2500 \\ 48^2 + 14^2 &= 2304 + 196 \\ &= 2500 \end{aligned}$$

12. Han will have to skate an additional 15 m before he reaches Josie.

$$\begin{aligned} 25^2 &= 20^2 + b^2 \\ b &= \sqrt{625 - 400} \\ b &= 15 \end{aligned}$$

13. To find the perimeter, the third side of the triangle must be found. To find the third side, the length of the line from side AC to vertex B must be found.

$$\begin{aligned} a^2 + (5\text{ cm})^2 &= (13\text{ cm})^2 \\ a &= \sqrt{169 - 25}\text{ cm} \\ a &= 12\text{ cm} \end{aligned}$$

The length of BC can be found now.

$$\begin{aligned} BC^2 &= (9\text{ cm})^2 + (12\text{ cm})^2 \\ BC &= \sqrt{81 + 144}\text{ cm} \\ BC &= 15\text{ cm} \end{aligned}$$

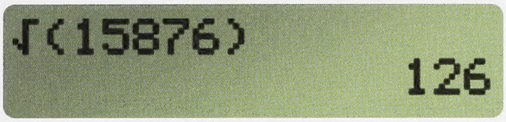
The perimeter of the triangle is 15 cm + 13 cm + 5 cm + 9 cm = 42 cm.

14. For the triangle to be a right triangle, it would have to satisfy the Pythagorean relationship. Since it does not, it is not a right triangle.

$$\begin{aligned} (12\text{ cm})^2 (12\text{ cm})^2 &= 288\text{ cm}^2 \\ (18\text{ cm})^2 &= 324\text{ cm}^2 \\ (18\text{ cm})^2 &\neq (12\text{ cm})^2 + (12\text{ cm})^2 \end{aligned}$$

15. a. For a number to be a perfect square, each prime factor must appear an even number of times. Since 2 appears twice, 3 appears four times, and 7 appears twice, the number 15 876 is a perfect square.

b. Using a calculator, your answer should look like the following:



c. Since 2 appears twice in the factorization of 15 876, 2 appears once as a factor of its square root. Since 3 appears four times in the factorization of 15 876, 3 appears twice as a factor of its square root. Since 7 appears twice in the factorization of 15 876, 7 appears once as a factor of its square root. The square root can be calculated as $2 \times 3 \times 3 \times 7 = 126$.